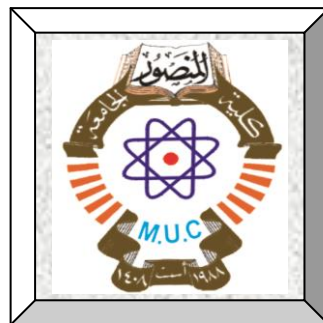


قسم  
هندسة اتصالات  
المرحلة الرابعة



الجامعة



المنصور



كلية

**Antenna**

**2017 – 2018**

**الهوائيات**

**P-1**

**Lec.**  
**Dr. Salman O. M.**

**4500**

**Subject: Antennas and Wave Propagation**

Th:2 hrs/w

Lab:-- hrs/w

**First Semester: Antennas**

**Ch. 1:Antenna concept and Parameters:**

(20 hrs)

- Fundamental parameters of antenna
- Basic antenna concepts.
- Radiation pattern.
- Field regions.
- Gain and directivity.
- Radiation resistance.
- Polarization.

**Ch. 2: Main types of antennas**

(20 hrs)

- Dipole antenna.
- Short dipole antenna.
- Thin linear antenna
- Loop antennas.
- Reflector antennas.
- Radiating aperture antennas.
- Antenna arrays.

**Second Semester: Antennas and Wave Propagation**

**Ch. 3: Smart antennas**

(6 hrs)

**Ch. 4: Wave Propagation**

(14hrs)

**Total Hrs: 60**

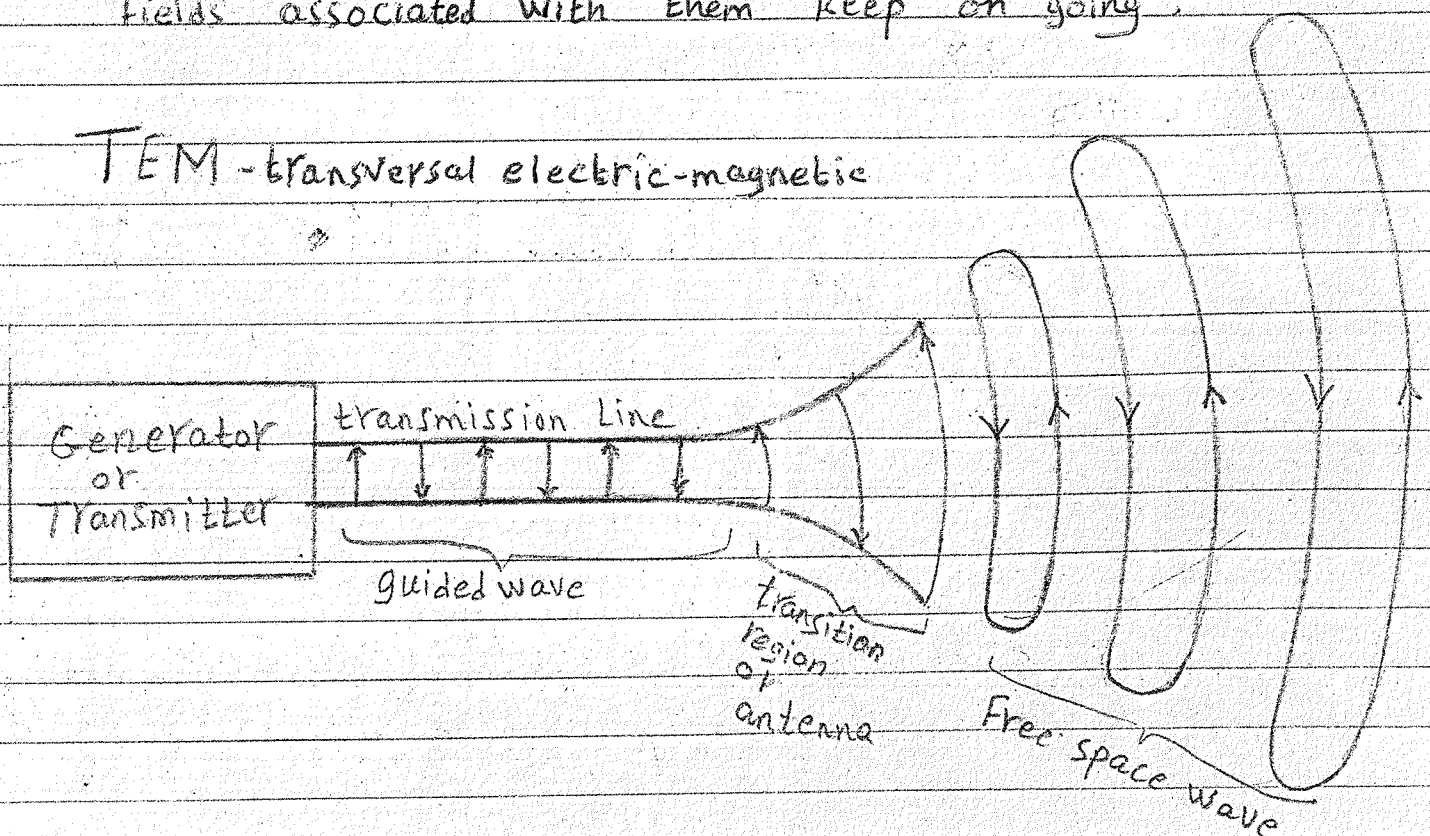
①

# ANTENNAS

The antenna is a transition device, or transducer, between a guided wave and a free-space wave or vice versa.

A two-wire transmission line is shown in figure below, connected to a radio-frequency generator (or transmitter). Along the uniform part of the line, energy is guided as a plane TEM-mode. The spacing between wires is assumed to be a small fraction of a wavelength. At the right, the transmission line is opened out. As the separation approaches the order of a wavelength or more, the wave tends to be radiated so that the opened-out line acts like an antenna which launches a free-space wave. The currents on the transmission line flow out on the antenna and end there, but the fields associated with them keep on going.

TEM-transversal electric-magnetic



②

## Fundamental parameters of antenna

### 1. Radiation pattern

An antenna radiation pattern or antenna pattern is defined as "a mathematical function or a graphical representation of the radiation properties of the antenna

as a function of space coordinates. In most cases, the radiation pattern is determined in the far-field region and is represented as a function of the directional coordinates. Radiation properties include power flux density, radiation intensity, field strength, directivity, ...

The radiation property of most concern is the two or three-dimensional spatial distribution of radiated energy "as a function of the observer's position along a path or surface of constant radius.

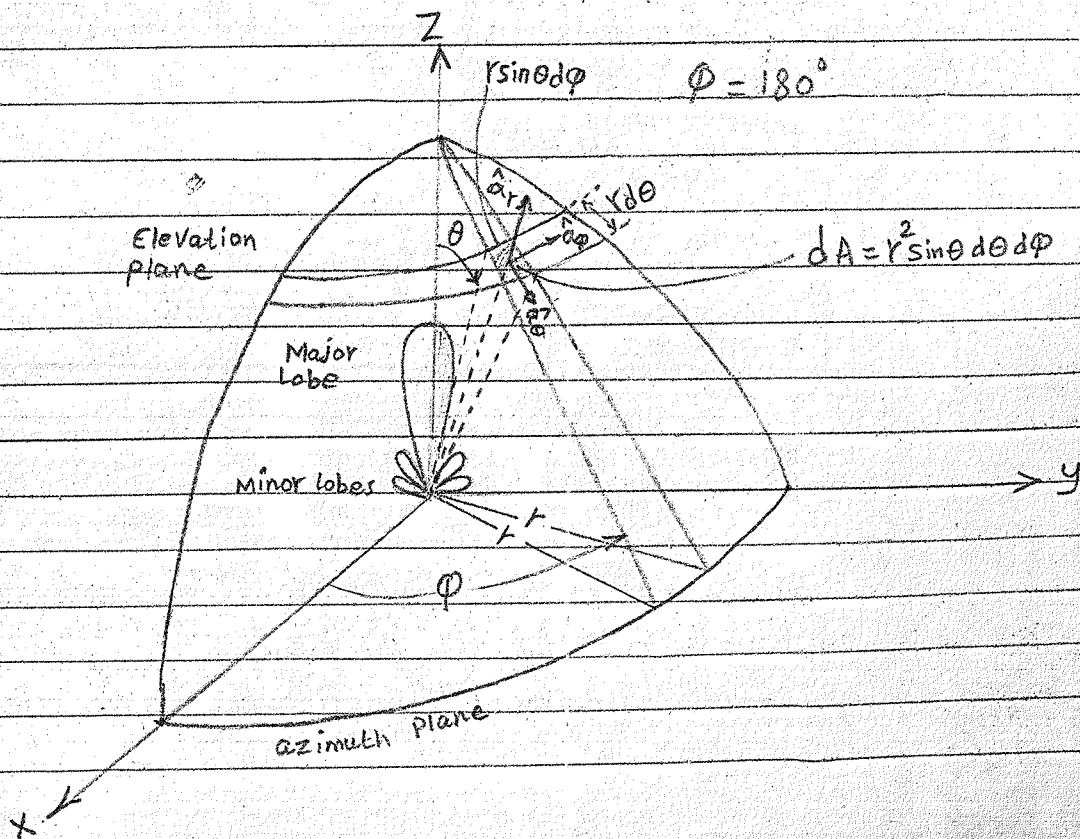
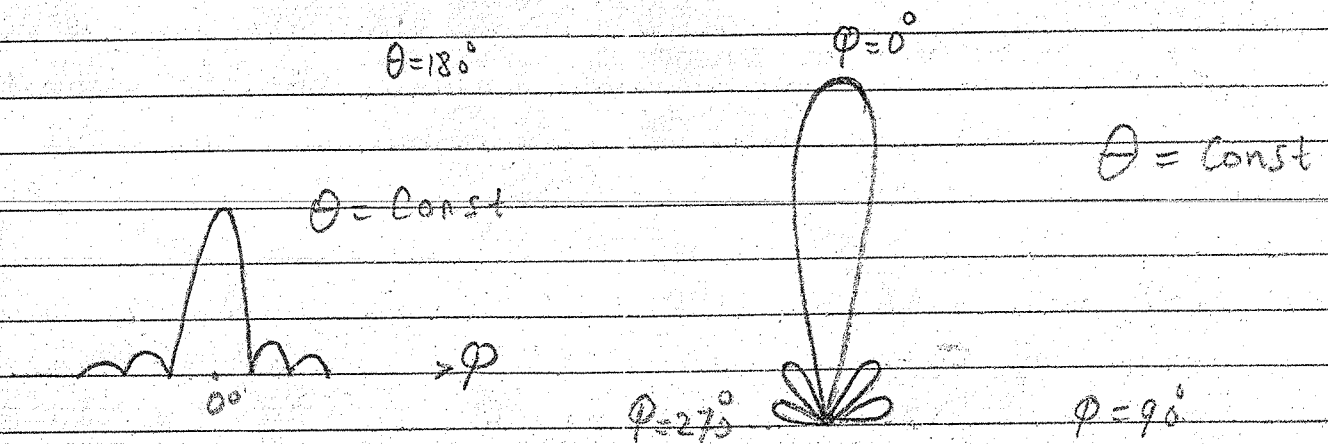
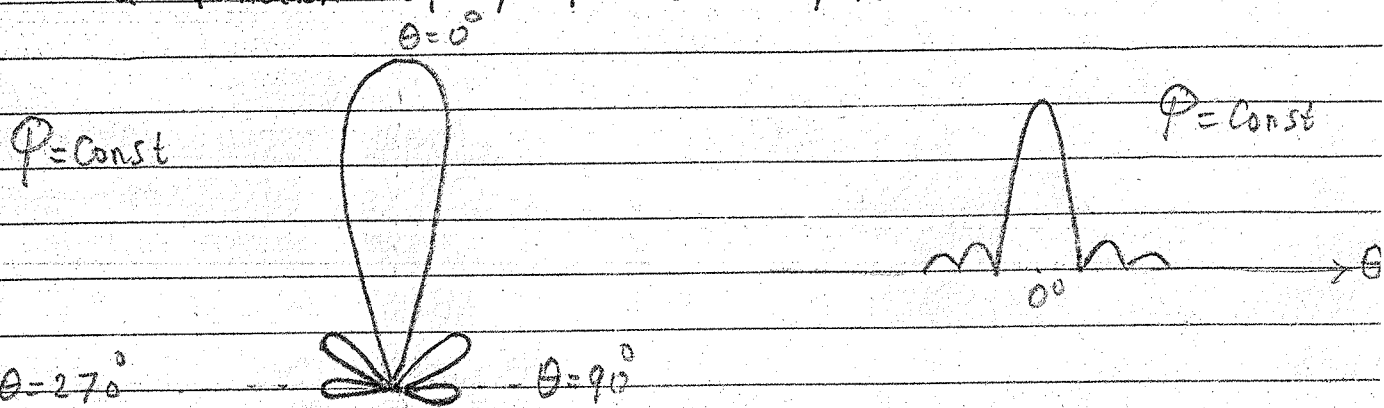
A convenient set of coordinates is shown in figure below. A trace of the received power at a constant radius is called the power pattern. On the other hand, a graph of the spatial variation of the electric (or magnetic) field along a constant radius is called an amplitude field pattern.

In ~~practice~~ practice, the three-dimensional pattern is measured and recorded in a series of two-dim



(3)

ensional pattern is measured ~~and recorded~~. However, for most practical application a few plots of the pattern as a function of  $\theta$  for some particular values of  $\phi$ , plus a few plots as a function of  $\phi$  for some particular values of  $\theta$ .



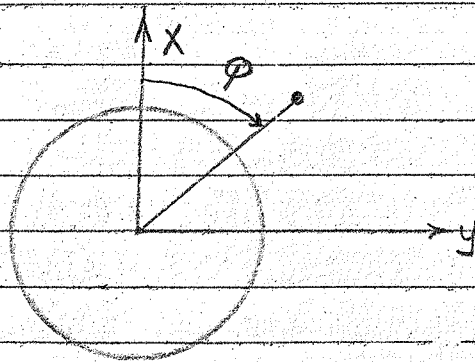
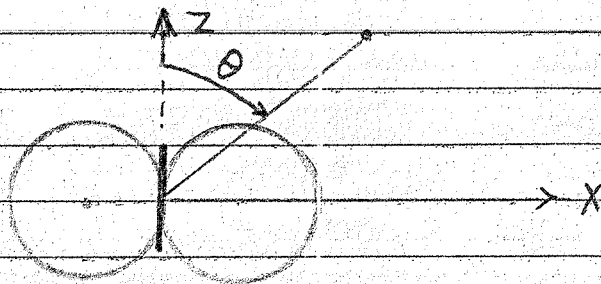
4

## 1. Isotropic, Directional and Omnidirectional patterns

- An isotropic radiator is defined as 'a hypothetical lossless antenna having equal radiation in all directions.'

- A directional antenna is one 'having the property of radiating or receiving electromagnetic waves more effectively in some direction than in others.'

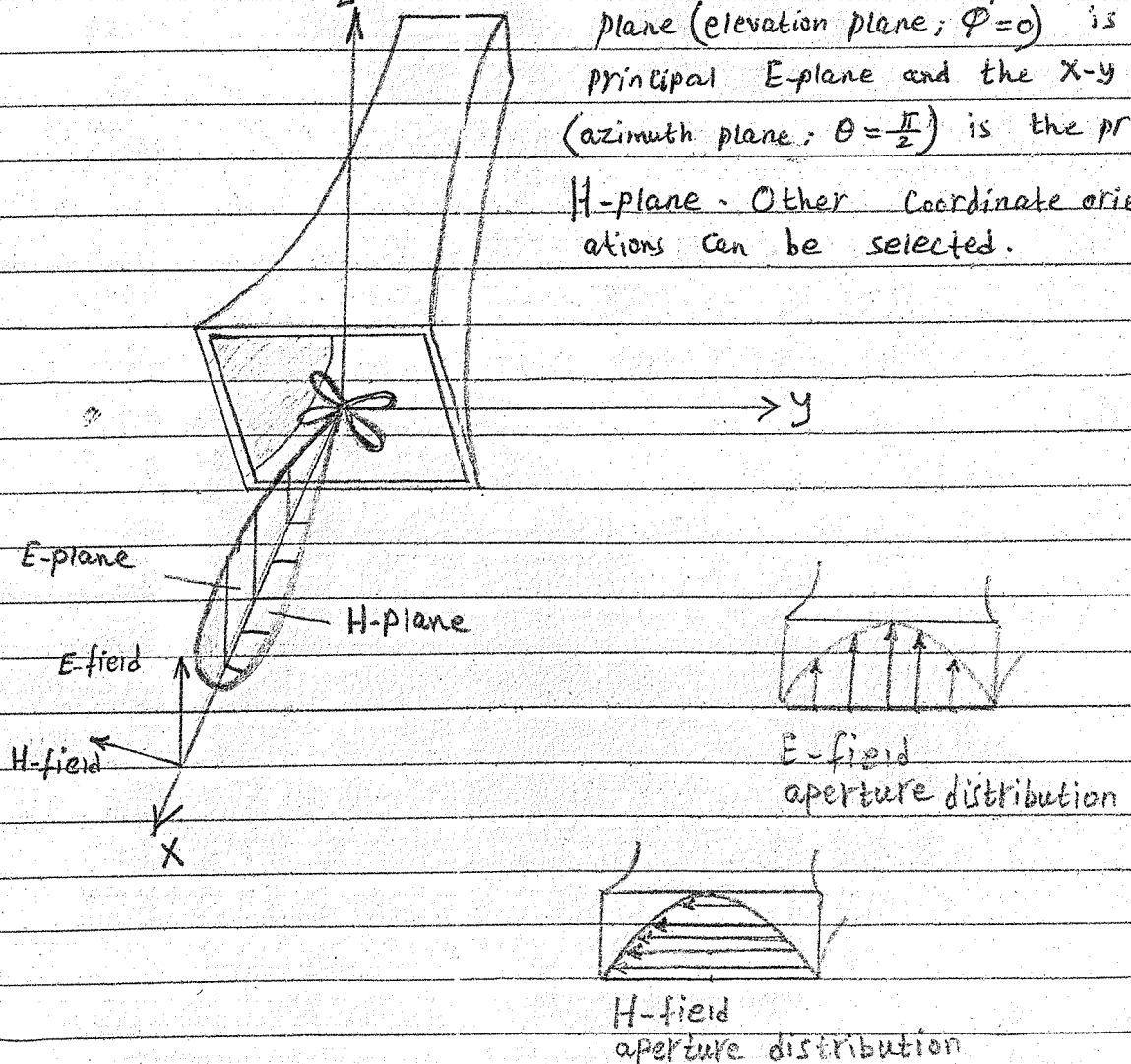
Omnidirectional antenna is defined as one 'having an essentially nondirectional pattern in a given plane (for example in azimuth plane) and a directional pattern in any orthogonal plane (in elevation plane). An omnidirectional pattern is then a special type of a directional pattern.



5

## 1.2- Principal patterns

For a linearly polarized antenna, performance is often described in terms of its principal E- and H-plane patterns. The E-plane is defined as 'the plane containing the electric-field vector and the direction of maximum radiation' and the H-plane as 'the plane containing the magnetic-field vector and the direction of maximum radiation'. Although it is very difficult to illustrate the principal patterns without considering a specific example, it is the usual practice to orient most antennas so that at least one of the principal plane patterns coincide with one of the geometrical principal planes. An illustration is shown in figure below. For this example, the x-z plane (elevation plane;  $\Phi=0$ ) is the principal E-plane and the x-y plane (azimuth plane;  $\Theta=\frac{\pi}{2}$ ) is the principal H-plane. Other coordinate orientations can be selected.



(10)

### 1.3- Radiation Pattern Lobes

Various part of a radiation pattern are referred to as lobes which may be subclassified into major or main, minor, side, and back lobe.

A radiation lobe is a "portion of the radiation pattern bounded by regions of relatively weak radiation intensity"

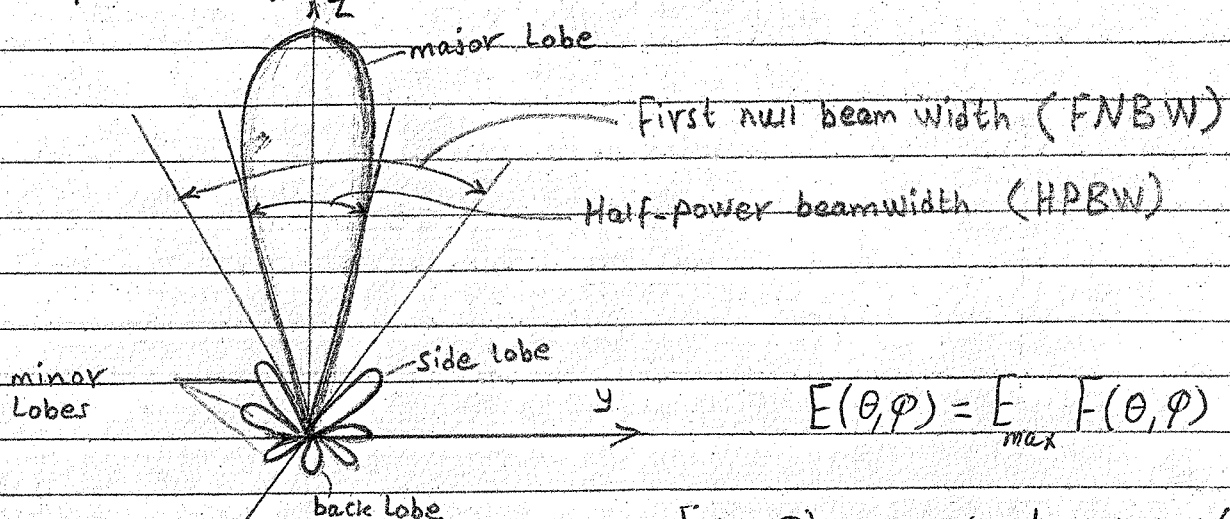
Figure below demonstrates a three-dimensional polar pattern with a number of radiation lobes. Some are of greater radiation intensity than others, but all are classified as lobes.

A major lobe (also called main beam) is defined as 'the radiation lobe containing the direction of maximum radiation!'

A minor lobe is any lobe except a major lobe.

A side lobe is adjacent to the main lobe and occupies the hemisphere in the direction of the main lobe.

A back lobe is a radiation lobe whose axis makes an angle of approximately  $180^\circ$  with respect to the beam of an antenna.



$$E(\theta, \phi) = E_{\max} F(\theta, \phi)$$

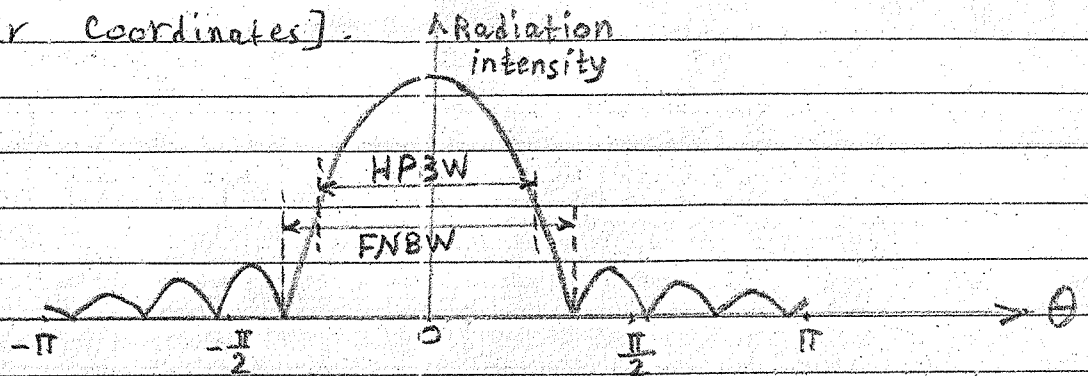
$F(\theta, \phi)$  - normalized value of electric field, or

$$F(\theta, \phi) = \frac{E(\theta, \phi)}{E_{\max}} \quad [\text{normalized field pattern}]$$

$$P_n(\theta, \phi) = \left( \frac{E(\theta, \phi)}{E_{\max}} \right)^2 - \text{normalized power pattern}$$

(7)

Figure below demonstrates a linear plot of power pattern and its associated lobes and beam widths [rectangular coordinates].



Minor Lobes usually represent radiation in undesired directions, and they should be minimized. Side lobes are normally the largest of the minor lobes. The level of the minor lobe is usually expressed as a ratio of the power density in the lobe in question to that of the major lobe. This ratio is often termed the side lobe ratio or side lobe level. Side lobe levels of -20 dB or smaller are usually not desirable in most applications.

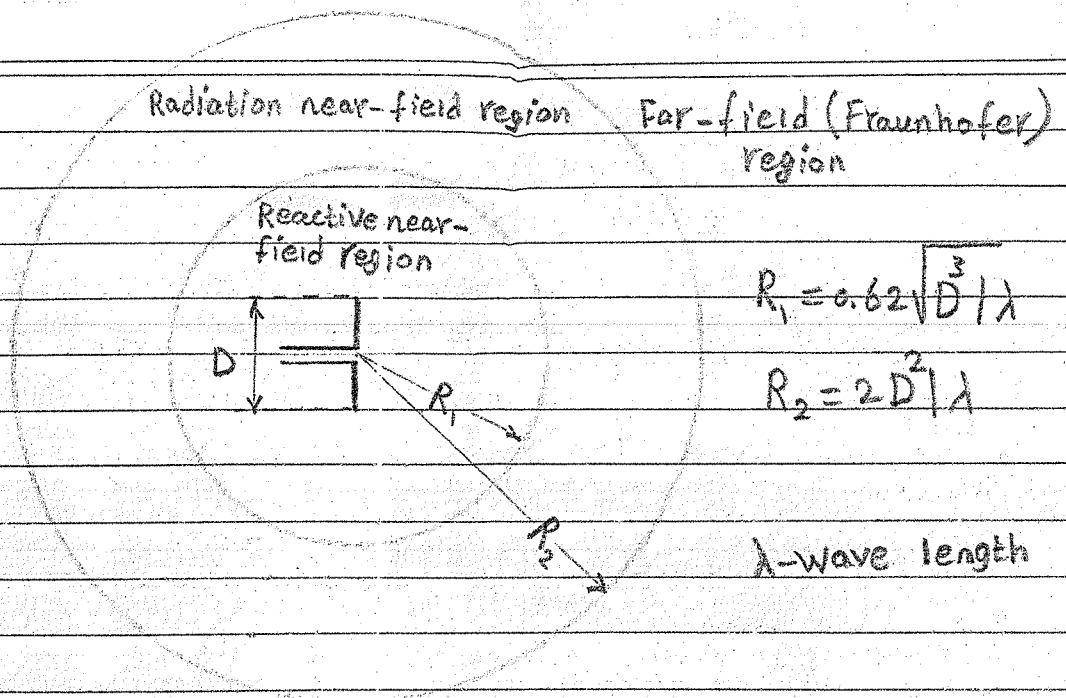
## Field Regions

The space surrounding an antenna is usually subdivided into three regions: (a) reactive near-field

- (b) radiating near-field (Fresnel) and
- (c) far-field (Fraunhofer) regions.

These regions are so designated to identify the field structure in each. Although no abrupt changes in the field configurations are noted as the boundaries are crossed, there are distinct differences among them.

8



- Reactive near-field region

$$R \approx 0.62 \sqrt{D^3 / \lambda}$$

for very short dipole, the out boundary is commonly taken to exist at a distance  $\lambda / 2\pi$  from the antenna.

The reactive field predominates in this region.

- Radiating near field (Fresnel)

$$0.62 \sqrt{D^3 / \lambda} \leq R \leq 2 D^2 / \lambda$$

The radiation field predominate and where <sup>the angular</sup> radiation field distribution is dependent upon the distance from the antenna.

If the antenna has a maximum dimension that is not large compared to the wavelength, this region may not exist.

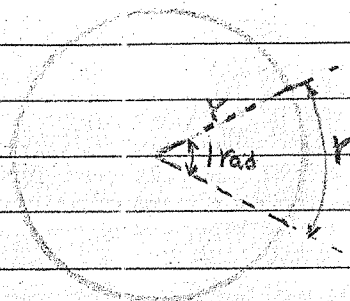
- Far-Field (Fraunhofer) region is defined as that region of the field of an antenna where the angular field distribution is essentially independent of the distance from the antenna.



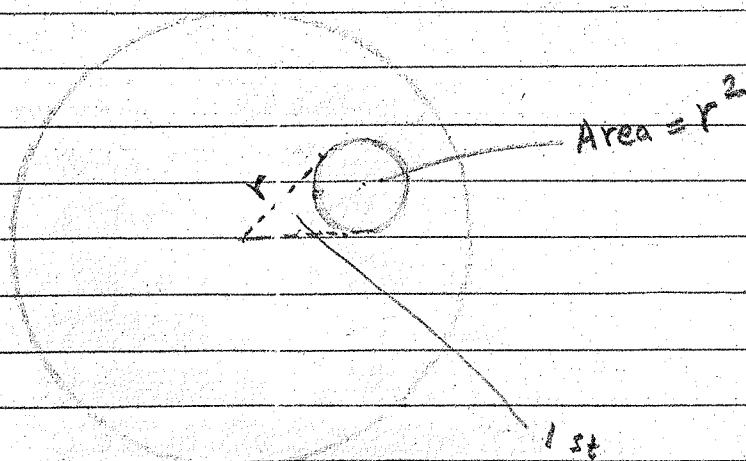
(9)

## Radian and Steradian

One radian is defined as the plane angle with its vertex at the center of a circle of radius  $r$  that is subtended by an arc whose length  $r$ .



The measure of a solid angle is a steradian. One steradian is defined as the solid angle with its vertex at the center of a sphere of radius  $r$  that is subtended by a spherical surface area equal to that of square with each side of length  $r$ .



## Radiation power density

The quantity used to describe the power associated with an electromagnetic wave is the instantaneous Poynting vector defined as

~~$S = E \times H$~~   $S = E \times H$

10

## Beam Solid angle

The beam solid angle ( $\Omega_A$ ) is given by the integral of the normalized power pattern over a sphere ( $4\pi$  sr),

or

$$\Omega_A = \iint_{4\pi} P_n(\theta, \varphi) d\Omega \quad (\text{sr})$$

where

$$P_n(\theta, \varphi) = \frac{S(\theta, \varphi)}{S(\theta, \varphi)_{\max}},$$

$S(\theta, \varphi)$  - Poynting vector (power density)

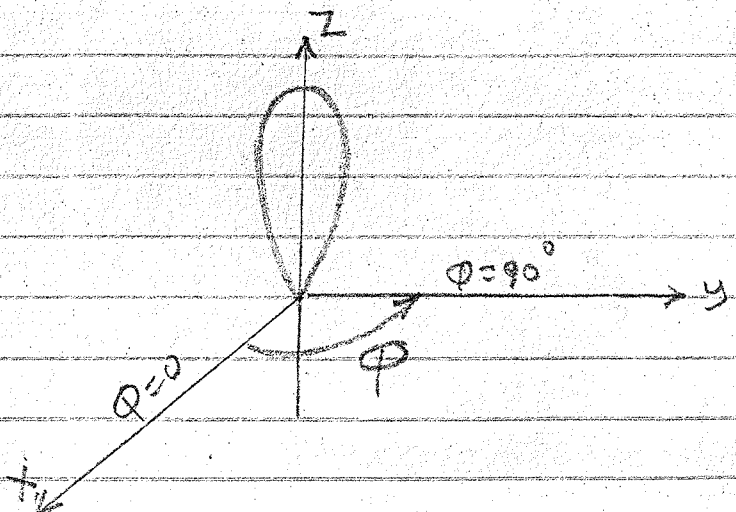
$$d\Omega = \sin\theta d\theta d\varphi$$

$$\Omega_A \approx \theta_{HP} \varphi_{HP}, \text{ where}$$

$\theta_{HP}$  - half power beam width in  $xz$ -plane ( $\varphi=0$ )

$\varphi_{HP}$  - half power beam width in  $yz$ -plane ( $\theta=90^\circ$ )

$\theta_{HP}, \varphi_{HP} [\text{rad}]$



11

# Directivity

The directivity ( $D$ ) of an antenna is given by the ratio of the maximum radiation intensity (power per unit solid angle)  $U(\theta, \phi)_{\max}$  to the average radiation intensity  $U_{AV}$  (average over a sphere) or at a certain distance from the antenna the directivity may expressed as the ratio of the maximum to the average pointing vector.

$$D = \frac{U(\theta, \phi)_{\max}}{U_{AV}} = \frac{4\pi U(\theta, \phi)_{\max}}{P} = \frac{4\pi (\text{maximum radiation intensity})}{\text{total power radiated}}$$

also directivity may be given by

$$D = \frac{S(\theta, \phi)_{\max}}{S_{AV}}, \quad S(\theta, \phi) - \text{pointing vector}$$

$$S_{AV} = \frac{k^2}{4\pi r^2} \iint_{4\pi} S(\theta, \phi) d\Omega = \frac{1}{4\pi} \int_0^{2\pi} \int_0^\pi S(\theta, \phi) d\Omega \quad (\text{W/m}^2)$$

Let us now develop a more general expression for the directivity. Let the radiation pattern be expressed as

$$U(\theta, \phi) = U_a f(\theta, \phi), \quad \text{where } U_a = \text{a constant}$$

$$U(\theta, \phi)_{\max} = U_a f(\theta, \phi)_{\max}$$

For the special case where

$$f(\theta, \phi) = 1$$

$$\text{then } U(\theta, \phi)_{\max} = U_a$$

$$f(\theta, \phi) = \frac{U(\theta, \phi)}{U(\theta, \phi)_{\max}}$$

$$F(\theta, \phi) = \frac{E(\theta, \phi)}{E(\theta, \phi)_{\max}}$$

$$f(\theta, \phi) = \frac{E^2(\theta, \phi)}{E^2(\theta, \phi)_{\max}}$$

(12)

~~Maximum value of  $f(\theta, \phi)$~~

$$U_{AV} = \frac{P}{4\pi} = \frac{\iint_{4\pi} U_a f(\theta, \phi) d\Omega}{4\pi},$$

Where  $P$  = total power radiated

$d\Omega = \sin\theta d\theta d\phi$  = element of solid angle

The directivity  $D$  is then given by

$$\begin{aligned} D &= \frac{U(\theta, \phi)_{\max}}{U_{AV}} = \frac{U_a f(\theta, \phi)_{\max}}{(\iint_{4\pi} U_a f(\theta, \phi) d\Omega) / 4\pi} = \frac{4\pi f(\theta, \phi)_{\max}}{\iint_{4\pi} f(\theta, \phi) d\Omega} \\ &= \frac{4\pi}{(\iint_{4\pi} f(\theta, \phi) d\Omega) / f(\theta, \phi)_{\max}} = \frac{4\pi}{\Omega_A}, \end{aligned}$$

where  $\Omega_A$  is defined as the beam area, or beam solid angle. It is given by

$$\begin{aligned} \Omega_A &= \frac{\iint_{4\pi} f(\theta, \phi) d\Omega}{f(\theta, \phi)_{\max}} = \iint_{4\pi} P_n(\theta, \phi) d\Omega = \int_0^{2\pi} \int_0^\pi P_n(\theta, \phi) d\Omega \\ &= \int_0^{2\pi} \int_0^\pi P_n(\theta, \phi) d\phi \sin\theta d\theta, \end{aligned}$$

$$\phi = 0 - 2\pi$$

$$\theta = 0 - \pi$$

$$D = \frac{S(\theta, \phi)_{\max}}{S_{AV}} = \frac{\frac{4\pi}{\int_0^{2\pi} \int_0^\pi \frac{S(\theta, \phi)}{S(\theta, \phi)_{\max}} d\Omega}}{\int_0^{2\pi} \int_0^\pi P_n(\theta, \phi) d\Omega} = \frac{4\pi}{\Omega_A}$$

Q. 13

$$D = \frac{U(\theta, \phi)_{\max}}{U_{AV}} = \frac{4\pi}{\Omega_A}$$

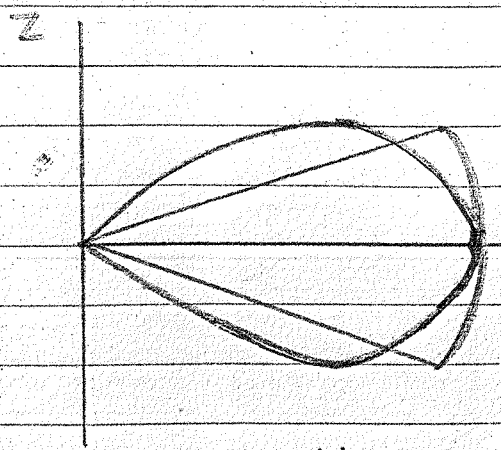
$$U_{AV} = \frac{P}{4\pi}, \quad P \text{ total power radiated}$$

$$4\pi U_{AV} = U(\theta, \phi)_{\max} \Omega_A$$

$$P = U(\theta, \phi)_{\max} \Omega_A$$

$$\Omega_A = \frac{4\pi U_{AV}}{U(\theta, \phi)_{\max}} \text{ square radians (solid angle)}$$

The solid angle (beam area)  $\Omega_A$  is the solid angle through which all the power radiated would stream if the power per unit solid angle equaled the maximum value  $U(\theta, \phi)_{\max}$  over the beam area.



$\theta_{HP}$  = half power beam width in  $\theta$  plane  
 $\phi_{HP}$  = half power beam width in  $\phi$  plane

The neglecting the effect of minor lobes, we have approximately

$$\Omega_A \approx \theta_{HP} \phi_{HP}$$

Directivity of isotropic antenna  $D=1$  because  $P_n(\theta, \phi)=1$

$$D = \frac{4\pi}{\int_0^{2\pi} d\phi \int_0^\pi \sin\theta d\theta} = \frac{4\pi}{2\pi [-\cos\pi + \cos 0]} = 1$$

14

Example

Calculate the directivity of an isotropic antenna (uniform radiation intensity in all directions)

Solution

$$P_n(\theta, \varphi) = 1 \quad \text{and} \quad \Omega_A = 4\pi$$

$$D = \frac{4\pi}{4\pi} = 1$$

Example

Calculate the directivity of a short dipole.

Solution

$$P_n(\theta, \varphi) = \frac{H_\varphi^2(\theta, \varphi)}{H_\varphi^2(\theta, \varphi)_{\max}} = \sin^2 \theta \quad ; \quad P_n(\theta, \varphi) = \frac{E_\theta^2(\theta, \varphi)}{E_\theta^2(\theta, \varphi)_{\max}} = \sin^2 \theta$$

$$D = \frac{4\pi}{\iint P_n(\theta, \varphi) d\Omega} = \frac{4\pi}{\int_0^{2\pi} \int_0^\pi \sin^3 \theta d\varphi d\theta} = 1.5$$

$$D = \frac{U(\theta, \varphi)_{\max}}{U_{\text{av}}} = \frac{S(\theta, \varphi)_{\max}}{S(\theta, \varphi)}$$

$$\int_0^{2\pi} \int_0^\pi \sin^3 \theta d\varphi d\theta = 2\pi \underbrace{\int_0^\pi \sin^3 \theta d\theta}_{\frac{4}{3}} = \frac{8\pi}{3}$$



15

Example: Antenna has a lobe with both HPBW =  $20^\circ$ , Find D.

Solution

$$D = \frac{4\pi(\text{sr})}{\Omega_A(\text{sr})} = \frac{41253}{\theta_{HP}^\circ \phi_{HP}^\circ} = \frac{41253}{20^\circ \times 20^\circ} = 103 = 20.1 \text{ dB}$$

1 steradian = 1 st = (Solid angle of sphere) /  $4\pi$

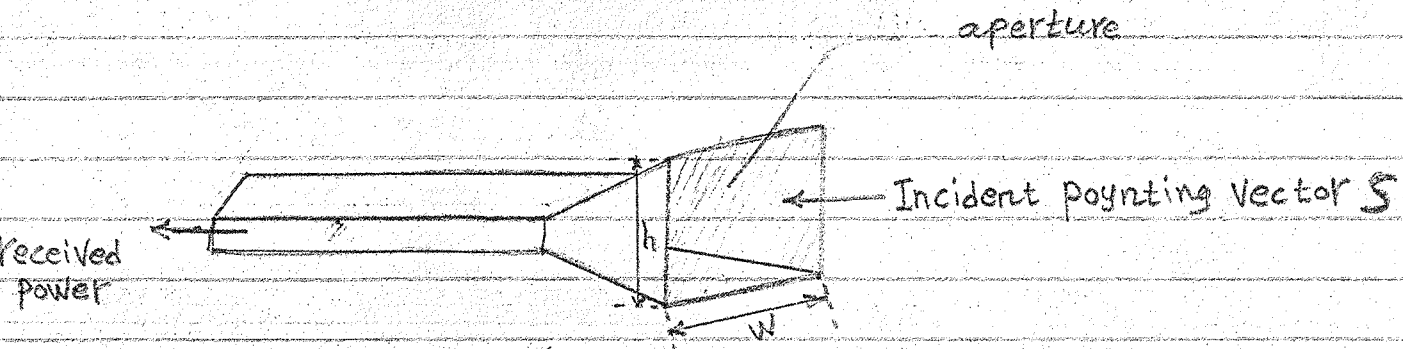
$$= 1 \text{ rad}^2 = \left(\frac{180}{\pi}\right)^2 (\text{deg})^2 = 3282.8064 \text{ square degrees}$$

$$4\pi \text{ steradians} = 3282.8064 \times 4\pi = 41252.96 \approx 41253 \text{ square degrees} \\ = \text{solid angle in a sphere}$$

$$D = \frac{4\pi}{\Omega_A(\text{sr})} = \frac{41253}{\theta_{HP}^\circ \phi_{HP}^\circ} = \frac{4\pi}{\theta_{HP}(\text{rad}) \phi_{HP}(\text{rad})} = \frac{4\pi}{20^\circ \times \frac{\pi}{180} \times 20^\circ \times \frac{\pi}{180}} = 103$$

Effective aperture

The effective aperture will be described with the antenna operating as a receiving device. Thus, if a plane wave with poynting vector  $S$  is incident normally on the aperture of a horn antenna.



The received power  $P$  is dependent on the collecting, or effective aperture of the antenna and is given by

$P = S A_e$ , from which the effective aperture is given by

$$A_e = \frac{P}{S} \text{ (m}^2\text{)},$$

$S$  = incident poynting vector  $[W/m^2]$

12

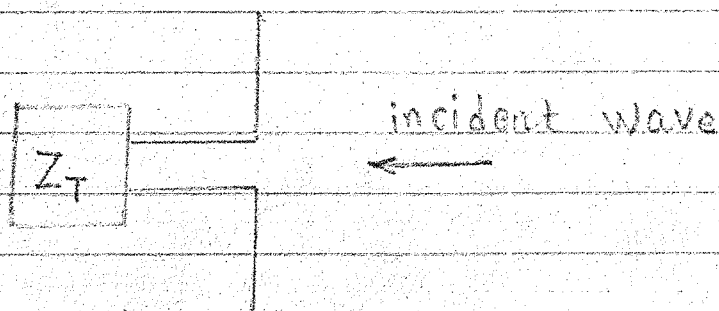
16

The effective aperture  $A_e$  represents the area over which power is extracted from the incident wave and delivered to the load.

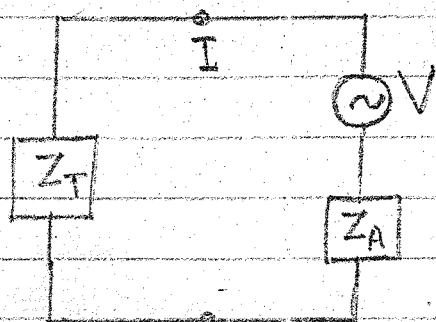
Effective aperture may be greater or less than the physical aperture, or cross section. [for horn antenna physical aperture "width  $W$  times height  $h$ "]

Also the effective aperture is used for linear antenna.

Consider a dipole receiving antenna situated in the field of a passing electromagnetic wave as suggested in figure:



The antenna collects power from the wave and delivers it to the terminating or load impedance  $Z_T$  connected to its terminals. The poynting vector, or power density of the wave, is  $S$  [watts per  $m^2$ ]. Referring to the equivalent circuit of figure below.



17

In general the terminating and antenna impedances are complex, thus

$$Z_T = R_T + jX_T$$

$$Z_A = R_A + jX_A$$

$$R_A = R_T + R_L$$

$R_L$  - Loss resistance

$R_r$  - radiation resistance

Let the power delivered by the antenna to the terminating impedance be  $P$ . Then

$$P = I^2 R_T$$

$$I = \frac{V}{\sqrt{(R_r + R_L + R_T)^2 + (X_A + X_T)^2}}$$

$$P = \frac{V^2 R_T}{(R_r + R_L + R_T)^2 + (X_A + X_T)^2}$$

Let us consider now the situation where the terminating impedance is the complex conjugate of the antenna impedance (terminal or load impedance matched to antenna) so that maximum power is transferred. Thus,

$$X_T = -X_A$$

$$R_T = R_r + R_L$$

$$P = \frac{V^2}{4R_r} = A S$$

18

If the antenna is Lossless ( $R_L = 0$ ) we obtain the maximum effective aperture

$$A_{em} = \frac{V^2}{4SR_r}$$

$$R_r = R_T, \quad R_L = 0$$

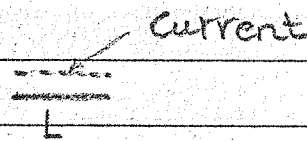
$A_{em}$  - maximum effective area.

### Example

Find the effective aperture of a short dipole with uniform current. Assume lossless short dipole ( $R_L = 0$ )

### Solution

$$L \ll \lambda$$



The emf induced in the short dipole is a maximum when the dipole is parallel to the incident electric field  $E$ . Hence

$$\text{emf} = V = E \cdot L, \quad E - \text{electric field intensity [V/m]}$$

$$S = \frac{E^2}{120\pi} \text{ [W/m}^2\text{]}$$

$L$  - Length of short dipole

$S$  - Poynting Vector

$$A_{em} = \frac{V^2}{4SR_r}, \quad R_r = 80\pi^2 \left(\frac{L}{\lambda}\right)^2$$

$$A_{em} = \frac{E^2 L^2}{4 \frac{E^2}{120\pi} \times 80\pi^2 \left(\frac{L}{\lambda}\right)^2} = 0.119 \lambda^2$$

In practice, losses are present because of finite conductivity of the dipole conductor [ $R_A = R_r + R_L$ ].

So that the actual aperture is less.

19

## Effective height

The effective height  $h$  (meters) of an antenna is another parameter related to the aperture.

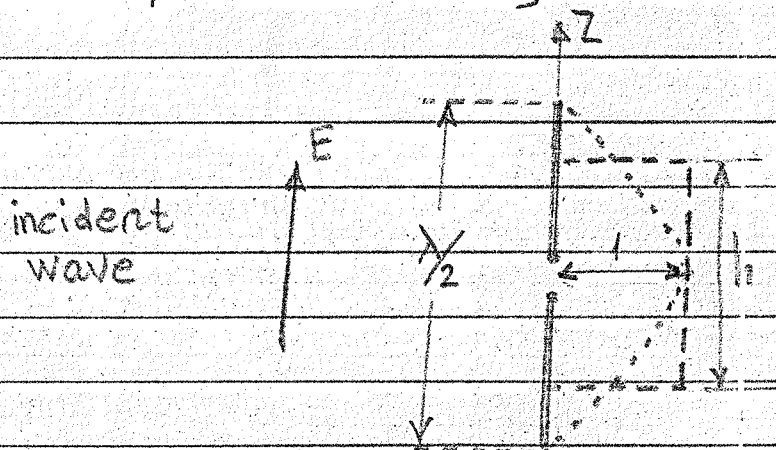
The induced voltage :

$$V = h E$$

$$h = \frac{V}{E}$$

effective height  $h$  = effective length  $L'$

Consider for example, a vertical dipole of length  $L = \frac{\lambda}{2}$  immersed in an incident field  $E$  of plane electromagnetic wave as shown in figure :



$$\begin{aligned} \int_0^{\lambda/2} \sin \frac{2\pi}{\lambda} z \cdot dz &= \int_0^{\lambda/2} \sin \frac{2\pi}{\lambda} z \cdot d\left(\frac{2\pi}{\lambda} z\right) / \frac{2\pi}{\lambda} = \frac{\lambda}{2\pi} \left[ \cos\left(\frac{2\pi}{\lambda} \frac{\lambda}{2}\right) + 1 \right] \\ &= \frac{\lambda}{2\pi} \times 2 = \frac{1}{\pi} \lambda = \frac{2}{\pi} \frac{\lambda}{2} = \frac{2}{\pi} L = 0.64 L \end{aligned}$$

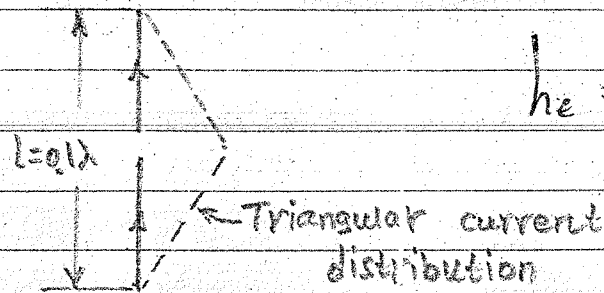
$$0.64 L = 1 \times h$$

$$h = 0.64 L = 0.64 \frac{\lambda}{2}$$

now consider another example :  $L = 0.1 \lambda$  (short dipole)



20



$$h_e = 0.5L \quad \left[ \begin{array}{l} L = 0.1\lambda \\ L \ll \lambda \end{array} \right]$$

The average current is  $\frac{1}{2}$  of the maximum so that the effective height is  $0.5L$

$$h = 0.5\lambda = 0.5 \times 0.1\lambda = 0.5L$$

For an antenna of radiation resistance  $R_r$  matched to its load, the power delivered to the load is equal to

$$P = \frac{1}{4} \frac{V^2}{R_r} = \frac{h_e^2 E^2}{4 R_r}$$

In terms of the effective aperture the same power is given by

$$P = S A_e = \frac{E^2 A_e}{Z_0} \text{ (W)}, \quad Z_0 = 120\pi = 377 \Omega$$

$$\frac{h_e^2 E^2}{4 R_r} = \frac{E^2 A_e}{377}$$

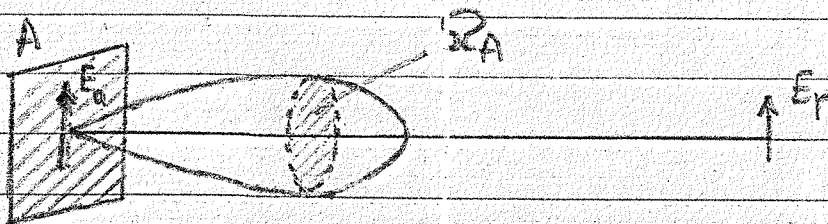
$$h_e = 2 \sqrt{\frac{R_r A_e}{377}} \text{ (m)}$$



21

## Effective aperture and directivity

Consider the electric field  $E_r$  at a large distance in a direction broadside to a radiation aperture as in figure



If the field intensity in the aperture is constant and equal to  $E_a$  the radiated power is given by

$$P = \frac{|E_a|^2}{Z_0} A, \quad Z_0 - \text{intrinsic impedance}$$

$A$  - antenna aperture

The power radiated may ~~be~~ also be expressed in terms of the field intensity  $E_r$  at a distance  $r$  by

$$P = \frac{|E_r|^2}{Z_0} r^2 \Omega_A, \quad \text{where } \Omega_A - \text{beam solid angle [sr]}$$

The field intensities  $E_r$  and  $E_a$  are related by

$$|E_r| = \frac{|E_a| A}{r \lambda}$$

$$P = \frac{|E_a|^2 A^2}{Z_0 r^2 \lambda^2} r^2 \Omega_A = \frac{|E_a|^2}{Z_0} A$$

$$\lambda^2 = \frac{A \Omega_A}{A}, \quad A = A_p, \quad \text{where } A_p \text{ is the physical aperture}$$

if the field is uniform over the aperture, as assumed, but in general  $A$  is the

maximum effective aperture  $A_{em}$  (losses equal zero), Thus

VIII  
(X)

22

$$\lambda^2 = A_{em} S_A$$

$$D = \frac{4\pi}{S_A} = \frac{4\pi}{\lambda^2} A_{em}$$

The gain

$$G = kD, \text{ where } k = A_e / A_{em}$$

$$G = \frac{4\pi}{\lambda^2} A_e$$

23

Effective aperture may be greater or less than the physical aperture, or cross section. [for horn antenna physical aperture - width  $W$  times height  $h$ ]

### Gain

The gain ( $G$ ) equals the directivity for Lossless antenna. If there is loss, the gain is less than the directivity. Thus, the gain  $G = kD$ , where

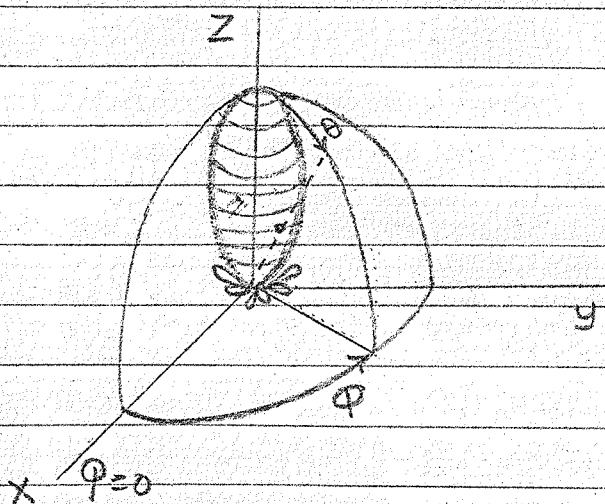
$k$  - efficiency factor ( $0 \leq k \leq 1$ ).

$$k = \frac{P_r}{P_r + P_L} = \frac{R_r}{R_r + R_L} = \eta, \quad P_L - \text{power of losses}$$

### Radiation Resistance

$k = \eta$  - efficiency of antenna

The power radiated by an antenna is given by the integral of the Poynting vector over a surface enclosing the antenna



$$\begin{aligned} \text{power radiated} &= \iint_{4\pi} S(\theta, \phi) d\Omega = S(\theta, \phi)_{\max} r^2 \iint_{4\pi} P_n(\theta, \phi) d\Omega \\ &= S(\theta, \phi)_{\max} r^2 \Omega_A \end{aligned}$$

24

From the conservation of power, the power

Radiated (for a lossless antenna) should be equal to the power input to the antenna or equal to

$I^2 R_r$ , where  $I$  is the current at the antenna terminals and  $R_r$  is the radiation resistance. Equating power in to power out,

$$I^2 R_r = S(\theta, \phi)_{\max} r^2 \Omega_A \quad (\text{W})$$

Thus the radiation resistance is seen to be a function of the pattern parameter  $\Omega_A$  as given by

$$R_r = \frac{S(\theta, \phi)_{\max} r^2}{I^2} \Omega_A \quad (\Omega)$$

### Example

: Define radiation resistance. Calculate the radiation resistance and efficiency of a ~~current element~~ short dipole ( $L = \frac{\lambda}{50}$ ) and loss resistance is 1.5 W.

### Solution

Radiation Resistance is that fictitious resistance which when connected in series with antenna, will consume the same amount of power as when ~~the antenna~~ actually radiating.

$$R_r = 80 \pi^2 \left( \frac{L}{\lambda} \right)^2 = 80 \pi^2 \left( \frac{\lambda/50}{\lambda} \right)^2 = 80 \pi^2 \left( \frac{1}{50} \right)^2 = 0.315 \Omega$$

$$\text{Efficiency} = \eta = \frac{R_r}{R_r + R_L} = \frac{0.315}{0.315 + 1.5} = 0.1739$$

$$\eta\% = 17.4$$

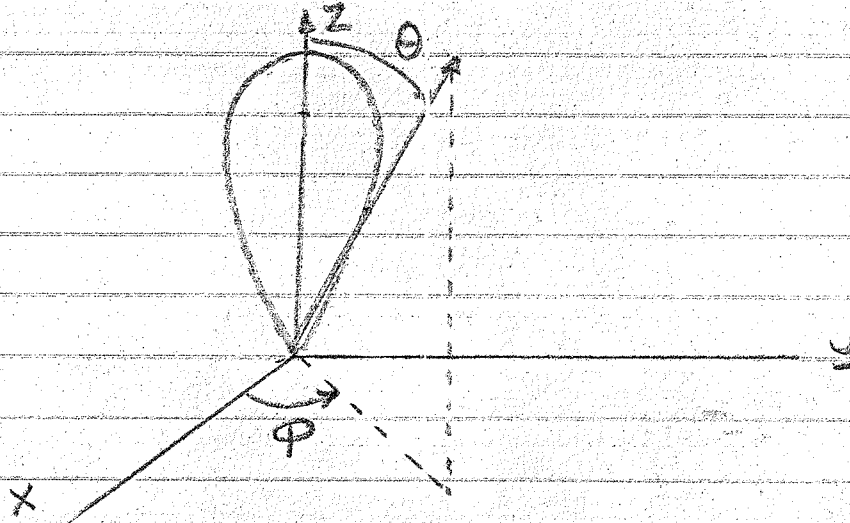
24

### Example

The radiation intensity of the antenna pattern can be adequately represented by

$$U = U_0 \cos \theta$$

Where  $U_0$  is the maximum radiation intensity. The radiation intensity exists only in the upper hemisphere ( $0 \leq \theta \leq \frac{\pi}{2}$ ,  $0 \leq \phi \leq 2\pi$ ), and it is shown in figure. Find the maximum directivity and compare it with its exact value.



### Solution

The half-power point of the pattern occurs at  $\theta = 60^\circ$ . Thus the beamwidth in the  $\theta$  direction is  $120^\circ$  or

$$\theta_{hp} = \frac{2\pi}{3}$$

$$\left[ \cos 60 = \frac{1}{2} \right]$$

Since the pattern is independent of the  $\phi$  coordinate, the beamwidth in the other plane is also equal to

$$\therefore \theta_{hp} = \frac{2\pi}{3}$$

$$D_0 = \frac{4\pi}{(2\pi/3)^2}$$

$$= \frac{9}{\pi} = 2.86$$

Now Let us find the exact value of the maximum directivity and compare the results.



24

$$U = U_0 \cos \theta$$

$$U_{\max} = U_0$$

$$\begin{aligned} P_{\text{rad}} &= \int_0^{2\pi} \int_0^{\pi/2} U_0 \cos \theta \sin \theta d\theta d\phi = 2\pi U_0 \int_0^{\pi/2} \cos \theta \sin \theta d\theta \\ &= \pi U_0 \int_0^{\pi/2} \sin(2\theta) d\theta = \pi U_0 \int_0^{\pi/2} \sin(2\theta) \frac{d(2\theta)}{2} \\ &= -\frac{\pi}{2} U_0 \cos 2\theta \Big|_0^{\pi/2} = -\frac{\pi}{2} U_0 [-1 - 1] = \pi U_0 \end{aligned}$$

$$D = \frac{4\pi U_{\max}}{P_r} = \frac{4\pi U_0}{\pi U_0} = 4$$

The exact directivity is 4 and its approximate value, is 2.86.

Example

Calculate the maximum effective aperture of an antenna which is operating at wavelength of two meters and a directivity of 100.

Solution

$$D = \frac{4\pi}{\lambda^2} A_e$$

$$A = \frac{D \lambda^2}{4\pi} = \frac{100 \times 2^2}{4\pi} = 31.84 \text{ m}^2$$



24

### Example

The radial Component of the radiated power density of a infinitesimal linear dipole of length  $L \ll \lambda$  is given by

$$S_r = \hat{a}_r A_0 \sin^2 \frac{\theta}{r^2} \quad (\text{W/m}^2)$$

$S_r$  - poynting Vector

where  $A_0$  is the peak value of the power density,  $\theta$  is the usual spherical coordinate, and  $\hat{a}_r$  is the radial unit vector.

Determine the maximum directivity of the antenna and express the directivity as a function of the directional angles  $\theta$  and  $\phi$ .

Solution: infinitesimal linear dipole mean very small

$$U = r^2 S_r = r^2 \frac{A_0 \sin^2 \theta}{r^2} = A_0 \sin^2 \theta$$

$$U_{\max} = A_0 \frac{r^2}{r^2} \{ \theta = 90^\circ \}$$

$$P_r = \iint U d\Omega = A_0 \int_0^{2\pi} \int_0^\pi \sin^2 \theta \sin \theta d\phi d\theta = A_0 \times 2\pi \int_0^\pi \sin^3 \theta d\theta$$

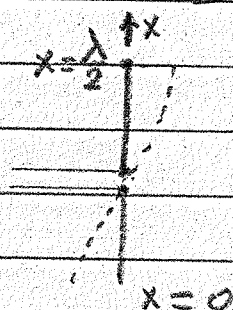
$$= A_0 \times 2\pi \times \frac{4}{3} = A_0 \frac{8\pi}{3} \text{ W}$$

$$D_{\max} = \frac{4\pi A_0}{A_0 \frac{8\pi}{3}} = \frac{4 \times 3 \pi A_0}{8 A_0 \pi} = \frac{3}{2} = 1.5$$

$$D = D_{\max} \sin^2 \theta$$

(24)<sup>IV</sup>Example

Calculate the maximum effective aperture of a  $\frac{\lambda}{2}$  (half-wave) dipole antenna.

Solution

$$dV = E \cdot dx \cos \frac{2\pi x}{\lambda}$$

$$\int dV = \int_0^{\lambda/2} E \cos \frac{2\pi x}{\lambda} dx = 2 \int_0^{\lambda/4} E \cos \frac{2\pi x}{\lambda} dx$$

$$= 2 \int_0^{\lambda/4} E \cos \frac{2\pi x}{\lambda} d\left(\frac{2\pi x}{\lambda}\right) \cdot \frac{\lambda}{2\pi} = \frac{2E\lambda}{2\pi} \left[ \sin \frac{2\pi x}{\lambda} - 0 \right]$$

$$= \frac{E\lambda}{\pi} \sin \frac{\pi}{2} = \frac{E\lambda}{\pi}$$

$$A_{\text{emax}} = \frac{V^2}{4SR_r} = \frac{E^2 \lambda^2}{\pi^2 \cdot 4 \cdot \frac{E^2}{1.2\pi} \cdot 73} = \frac{\lambda^2 \cdot 120}{\pi \cdot 4 \cdot 73} = \frac{\lambda^2 \cdot 30}{73\pi} = 0.13 \lambda^2$$

= 10.0

Example

Determine the maximum effective aperture of a beam antenna having (HPBW) of  $30^\circ$  and  $35^\circ$  in perpendicular planes intersecting in the beam axis.

Solution

$$D = \frac{41253}{\theta_E \times \theta_H} = \frac{41253}{30 \times 35} = \frac{41253}{1050}$$

$$D = \frac{4\pi A_{\text{emax}}}{\lambda^2}$$

$$A_{\text{emax}} = \frac{D \lambda^2}{4\pi} = \frac{41000 \lambda^2}{1050 \times 4\pi} = 3.1 \lambda^2$$

(24)<sup>V</sup>Example

An antenna has a field pattern given by  $E(\theta) = \cos^3 \theta$  for  $0 \leq \theta \leq 90^\circ$ . Find the beam area of this pattern.

Solution

$$P_n(\theta, \varphi) = \left[ \frac{E(\theta, \varphi)}{E(\theta, \varphi)_{\max}} \right]^2 = \left[ \frac{E(\theta)}{E(\theta)_{\max}} \right]^2 = \frac{E^2(\theta)}{E_{\max}^2} = \cos^4 \theta$$

$$E(\theta) = \cos^3 \theta$$

$$E(\theta)_{\max} = 1, \quad E(\theta) = \cos^3 \theta$$

$$\Omega_A = \int_0^{2\pi} \int_0^{\pi/2} \cos^4 \theta \sin \theta \, d\theta \, d\varphi = 2\pi \int_0^{\pi/2} \cos^4 \theta \sin \theta \, d\theta$$

$$= -2\pi \left[ \frac{1}{5} \cos^5 \theta \right]_0^{\pi/2} = -\frac{2\pi}{5} [\cos^5 90^\circ - \cos^5 0] = -\frac{2\pi}{5} [0 - 1]$$

$$= \frac{2\pi}{5} \text{ sr.}$$

Example

An antenna has a field pattern given by  $E(\theta) = \cos^2 \theta$  for  $0 \leq \theta \leq 90^\circ$ .

Find the half-power beamwidth (HPBW)

Solution

$E(\theta)$  at half power = 0.707. Thus  $0.707 = \cos^2 \theta$

$$\text{So } \cos \theta = \sqrt{0.707} \text{ and } \theta = 33^\circ$$

$$\text{HPBW} = 2\theta = 66^\circ$$

Example

In a microwave communication link, two identical antenna operating at 10 GHz are used with power gain of 40 dB. If the transmitter power is 1W, find the received power, if the range of the link is 30 km.

Solution

$$G_t = G_r = G = 40 \text{ dB}$$

$$\frac{P_t G_t}{4\pi R^2} \times A_r = P_r$$

$$40 = 10 \log G$$

$$G^{10} = 10^{40}$$

$$G = 10^4$$

$$G_r = \frac{4\pi}{\lambda^2} A_r, \quad \lambda = \frac{3 \times 10^8}{10 \times 10^9} = \frac{3}{100} \text{ m}$$

$$A_r = \frac{G_r \lambda^2}{4\pi} = \frac{10^4 \left(\frac{3}{100}\right)^2}{4\pi} = \frac{9}{4\pi} \text{ m}^2$$

$$P_r = \frac{1 \times 10^4}{4\pi \times (30 \times 10^3)^2} \times \frac{9}{4\pi} = \frac{9 \times 10^4}{16\pi^2 \times 9 \times 10^8} = \frac{1}{16\pi^2 \times 10^4} = 0.63 \mu\text{W}$$

$P_t$  - transmitted power

$P_r$  - received power

(25)

# Polarization

Polarization of an antenna in a given direction is defined as "the polarization of the wave transmitted (radiated) by the antenna."

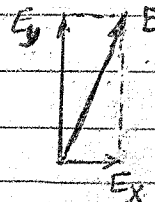
— Linear polarization — A time harmonic wave is linearly polarized at a given point in space if the electric field (or magnetic field) vector at that point is always oriented along the same straight line at every instant of time. This is accomplished if the field vector (electric or magnetic) possesses:

a. Only one component



$$E_y = E_1 \sin(\omega t - \beta z)$$

b. Two orthogonal linear components that are in time phase or  $180^\circ$  (or multiples of  $180^\circ$ ) out of phase.



$$\Delta\phi = \phi_y - \phi_x = n\pi, n = 0, 1, 2, 3, \dots$$

## Circular Polarization

A time harmonic wave is circularly polarized at a given point in space if the electric (or magnetic) field vector at that point traces a circle as a function of time.

The necessary and sufficient conditions to accomplish this are if the field vector possesses all of the following:

- (a) The field must have two orthogonal linear components, and
- (b) The two components must have the same magnitude, and
- (c) The two components must have a time-phase difference of odd multiples of  $90^\circ$ .



Circular polarization can be achieved only when the magnitudes of the two components are the same and the time-phase difference between them is odd multiples of  $\pi/2$ . That is

$$E_1 = E_2$$

$$\Delta\phi = \begin{cases} +(\frac{1}{2} + 2n)\pi, & n=0, 1, 2, 3, \dots \text{CW} \\ -(\frac{1}{2} + 2n)\pi, & n=0, 1, 2, 3, \dots \text{CCW (Counter clockwise)} \end{cases}$$

for  $n=0$ :

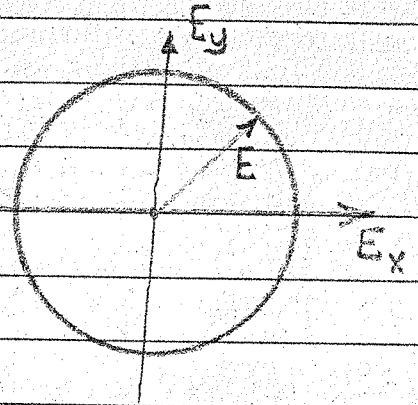
$$E_x = E_1 \sin(\omega t - \beta z)$$

$$E_y = E_1 \sin(\omega t - \beta z + 90^\circ)$$

for  $z=0$ :

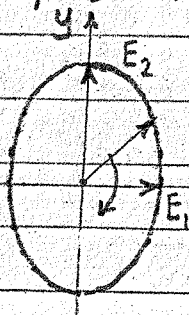
$$E_x = E_1 \sin \omega t$$

$$E_y = E_1 \sin(\omega t + 90^\circ)$$



Elliptical polarization can be achieved only when the

time-phase difference between the two components is odd multiples of  $\frac{\pi}{2}$  and their magnitudes are not the same or when the time-phase difference between the two components is not equal to multiples of  $\frac{\pi}{2}$  (irrespective of their magnitudes). That is



$$\Delta\phi = \phi_y - \phi_x = \begin{cases} +(\frac{1}{2} + 2n)\pi & \text{for CW [clockwise]} \\ -(\frac{1}{2} + 2n)\pi & \text{for CCW} \end{cases}$$

$$n=0, 1, 2, 3, \dots$$

$$E_2 \neq E_1$$

for  $n=0$ :

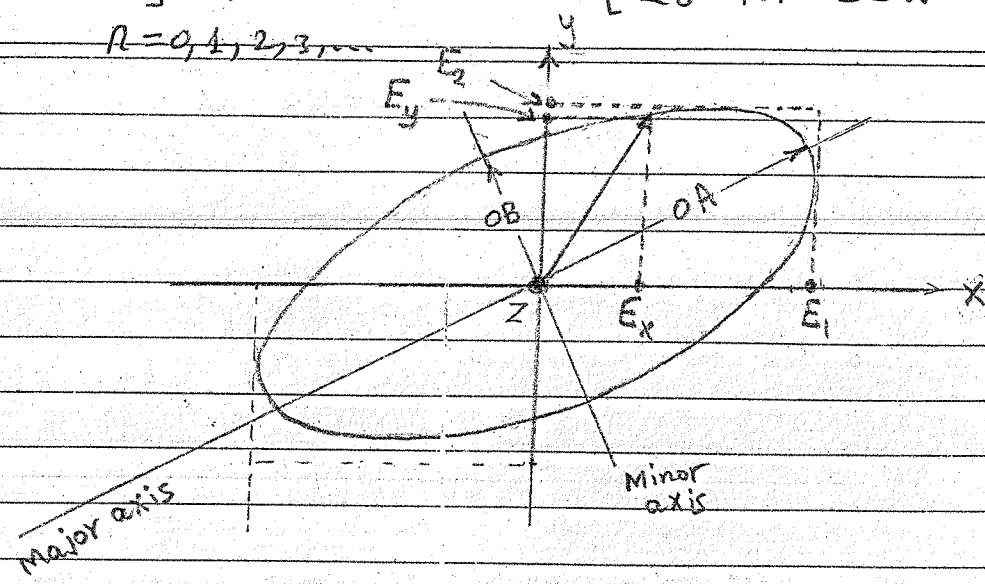
$$E_x = E_1 \sin(\omega t - \beta z)$$

$$E_2 \neq E_1$$

$$E_y = E_2 \sin(\omega t - \beta z + 90^\circ)$$



(27) or  $\Delta\varphi = \varphi_y - \varphi_x = \pm n\pi/2 = \begin{cases} > 0 \text{ for CW} \\ < 0 \text{ for CCW} \end{cases}$   
 $n = 0, 1, 2, 3, \dots$



A time-harmonic wave is elliptically polarized if the tip of the field vector (electric or magnetic) traces an elliptical locus in space. At various instants of time the field vector changes continuously with time at such a manner as to describe an elliptical locus. It is right-hand (clockwise) elliptically polarized if the field vector rotates clockwise, and it is the left-hand (counterclockwise) elliptically polarized if the field vector of the ellipse rotates counterclockwise.

## Polarization Loss factor

In general, the polarization of the receiving antenna will not be the same as the polarization of the incoming (incident) wave. This is commonly stated as "polarization mismatch". The amount of power extracted by the antenna from the incoming signal will not be maximum because of the polarization loss. Assuming that the electric field of the incoming wave can be written as

$$\mathbf{E}_i = \hat{\mathbf{p}}_w E_i$$

Where  $\hat{\mathbf{p}}$  is the unit vector of the wave, and the polar-

(28)

antenna can be expressed as

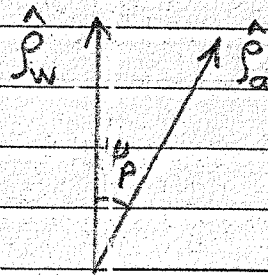
$$E_a = \hat{p}_a E_a$$

Where  $\hat{p}_a$  is its unit vector (polarization vector).

The polarization Loss can be taken into account by introducing a polarization Loss factor (PLF)

$$PLF = |\hat{p}_w \cdot \hat{p}_a|^2 = |\cos \psi_p|^2$$

(dimensionless)



Example

The electric field of a linearly polarized electromagnetic wave given by  $E_i = \hat{a}_x E_0(x, y) e^{j\beta z}$  is incident upon a linearly polarized antenna whose electric field polarization can be expressed as

$$E_a = (\hat{a}_x + \hat{a}_y) E_0(r, \theta, \phi)$$

Find the polarization Loss factor (PLF).

Solution

For the incident wave  $\hat{p}_w = \hat{a}_x$

and for the antenna

$$\hat{p}_a = \frac{1}{\sqrt{2}} (\hat{a}_x + \hat{a}_y)$$



The PLF is then equal to

$$PLF = |\hat{p}_w \cdot \hat{p}_a|^2 = |\hat{a}_x \cdot \frac{1}{\sqrt{2}} (\hat{a}_x + \hat{a}_y)|^2 = \frac{1}{2}$$

which in db is equal to

$$PLF = 10 \log \frac{1}{2} = -3 \text{ db}$$

(29)

Since the divergence of  $B$  is always zero.

$$[\nabla \cdot B = 0]$$

Therefore,  $B$  can be expressed as the curl of of sum other vector function. Let us designate this other vector function by  $A$ . Then

$$B = \nabla \times A$$

The function  $A$  is called the vector potential.

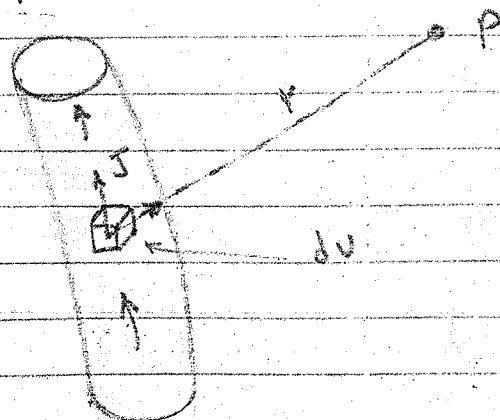
since  $\nabla \cdot B = 0$ , also

$$\nabla \cdot A = 0$$

$$, \quad \nabla \cdot (\nabla \times A) = 0$$

$$A = \frac{\mu}{4\pi} \iiint \frac{J}{r} dv,$$

Where  $J$  - current density at each volume element  $dv$  and  $r$  is the distance from each volume element to the point  $P$ .



~~Length of wire~~,  
 $L$  - Length of wire

$$J [A/m^2]$$

According to Maxwell's equation derived from Faraday's Law, we note, that the electric field is not zero but is equal to the time rate of decrease of  $B$ . Thus

$$\nabla \times E = - \frac{\partial B}{\partial t}$$

Since  $\nabla \times E$  is not zero, the relation  $E = -\nabla V$  is not sufficient for time-varying fields. An additional term is required. This may be found as follows: Since  $B = \nabla \times A$

for static electric field  $\nabla \times E = 0$

$$E = -\nabla V$$

$$\nabla \cdot (\nabla \times E) = 0$$

$$\nabla \times E = - \frac{\partial (\nabla \times A)}{\partial t}$$

from which

$$\nabla \times \left( E + \frac{\partial A}{\partial t} \right) = 0$$

Since the curl expression in parentheses in last equation equals zero, it must be equal to the gradient of a scalar function. Thus we can write

$$E + \frac{\partial A}{\partial t} = \nabla f, \text{ where } f \text{ is a scalar function.}$$

Let  $f = -V$ ,  $V$  - electric potential

For static fields this reduces to  $E = -\nabla V$ . In general case, where the field may vary with time

30

$E$  is given by both a scalar potential  $V$  and a vector potential  $A$ .

$$E = -\frac{\partial A}{\partial t} - \nabla V$$

If the time variation is harmonic we give,

$$E = -\nabla V - j\omega A \quad [V/m]$$

$$\left[ \begin{array}{l} V = \frac{1}{4\pi\epsilon_0} \int_V \frac{\rho}{r} dv \quad [V] \\ A = \frac{\mu}{4\pi} \int_V \frac{J}{r} dv \quad [Wb/m] \end{array} \right.$$

$$\left. \begin{array}{l} E e^{j\omega t} \\ V e^{j\omega t} \\ A e^{j\omega t} \end{array} \right\}$$

$$B = \nabla \times A \quad [Wb/m^2], \quad Wb - \text{Weber}$$

$\rho$  - volume charge density  $[C/m^3]$

$J$  - Volume current density  $[A/m^2]$

Wb - Weber

30

$$\Psi = \int_S \mathbf{B} \cdot d\mathbf{s} = \int_S (\nabla \times \mathbf{A}) \cdot d\mathbf{s} = \oint_L \mathbf{A} \cdot d\mathbf{L}$$

or  $\Psi = \oint_L \mathbf{A} \cdot d\mathbf{L}$

$\mathbf{A}$  - vector potential  $[\frac{\text{Wb}}{\text{m}}]$

$\mathbf{B}$  - magnetic flux density  $[\frac{\text{Wb}}{\text{m}^2}]$

If  $\mathbf{J} = 0$

$\mathbf{H} = -\nabla V_m$ ,  $V_m$  - magnetic scalar potential [ampere]

$$\mathbf{J} = \nabla \times \mathbf{H} = \nabla \times (-\nabla V_m) = 0$$

$$\nabla \times \mathbf{H} = \mathbf{J}$$

$$\frac{\partial D}{\partial t} = 0$$

### Example

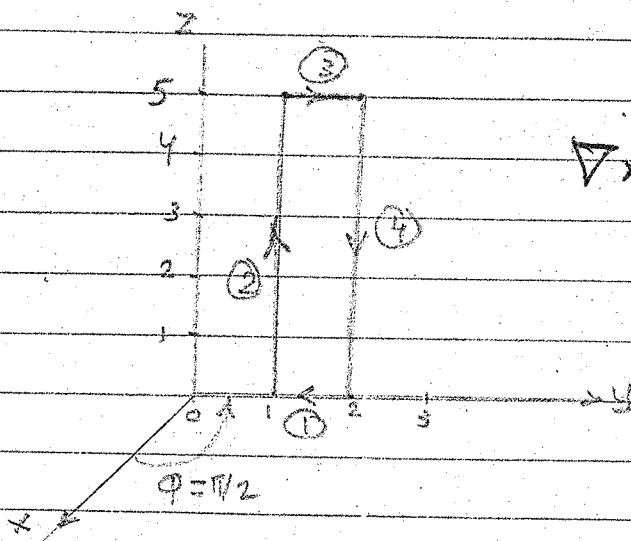
Given the magnetic vector potential  $\mathbf{A} = -\frac{\rho^2}{4} \mathbf{a}_z$   $\frac{\text{Wb}}{\text{m}}$ , calculate the total magnetic flux crossing the surface  $\phi = \frac{\pi}{2}$ ,  $1 \leq \rho \leq 2 \text{ m}$ ,  $0 \leq z \leq 5 \text{ m}$ .

### Solution

method-1

$$\mathbf{B} = \nabla \times \mathbf{A} = -\frac{\partial A_z}{\partial \rho} \mathbf{a}_\phi = -\frac{\rho}{2} \mathbf{a}_\phi$$

$$d\mathbf{s} = d\rho dz \mathbf{a}_\phi$$



$$\begin{aligned} \nabla \times \mathbf{A} &= \begin{vmatrix} \mathbf{a}_\rho & \mathbf{a}_\phi & \mathbf{a}_z \\ \frac{\partial}{\partial \rho} & 0 & 0 \\ 0 & -\frac{\rho}{2} & 0 \end{vmatrix} \\ &= \frac{\partial}{\partial \rho} \left( -\frac{\rho^2}{4} \right) \mathbf{a}_\phi = -\frac{\rho}{2} \mathbf{a}_\phi \end{aligned}$$

$$\Psi = \int \mathbf{B} \cdot d\mathbf{s} = \frac{1}{2} \int_{z=0}^5 \int_{\rho=1}^2 \rho d\rho dz = \frac{1}{4} \rho^2 \Big|_1^2 \Big|_0^5 = \frac{15}{4} = 3.75 \text{ Wb}$$

$$\left[ \nabla = \frac{\partial}{\partial \rho} \mathbf{a}_\rho + \frac{1}{\rho} \frac{\partial}{\partial \phi} \mathbf{a}_\phi + \frac{\partial}{\partial z} \mathbf{a}_z \right], \quad \nabla = \frac{\partial}{\partial \rho} \mathbf{a}_\rho + 0 + 0, \quad \mathbf{A} = 0 + 0 + \frac{\rho^2}{4} \mathbf{a}_z$$



III  
30

method 2

$$\Psi = \oint_L \mathbf{A} \cdot d\mathbf{L} = \Psi_1 + \Psi_2 + \Psi_3 + \Psi_4$$

since  $\mathbf{A}$  has only a  $z$ -component,

$$\Psi_1 = 0 = \Psi_3$$

That is

$$\begin{aligned} \Psi = \Psi_2 + \Psi_4 &= -\frac{1}{4} \left[ (1)^2 \int_0^5 dz + (2)^2 \int_5^0 dz \right] = -\frac{1}{4} \left[ 5 - \frac{4 \cdot 3}{6} \right] = -\frac{1}{4} [-15] = \frac{15}{4} \text{ wb} \\ &= 3.75 \text{ wb} \end{aligned}$$

$$\nabla = \frac{\partial}{\partial x} a_x + \frac{\partial}{\partial y} a_y + \frac{\partial}{\partial z} a_z - \text{Cartesian}$$

$$\nabla = \frac{\partial}{\partial \rho} a_\rho + \frac{1}{\rho} \frac{\partial}{\partial \phi} a_\phi + \frac{\partial}{\partial z} a_z - \text{cylindrical}$$

~~$$\frac{\partial}{\partial x} = \cos \phi \frac{\partial}{\partial \rho} - \frac{\sin \phi}{\rho} \frac{\partial}{\partial \phi}$$~~

$$\frac{\partial}{\partial x} = \cos \phi \frac{\partial}{\partial \rho} - \frac{\sin \phi}{\rho} \frac{\partial}{\partial \phi}$$

$$\nabla = a_r \frac{\partial}{\partial r} + a_\theta \frac{\partial}{\partial \theta} \frac{1}{r} + a_\phi \frac{1}{r \sin \theta} \frac{\partial}{\partial \phi} - \text{spherical}$$

$$\frac{\partial}{\partial x} \sin \theta \cos \phi \frac{\partial}{\partial r} + \frac{\cos \theta \cos \phi}{r} \frac{\partial}{\partial \theta} - \frac{\sin \phi}{\rho} \frac{\partial}{\partial \phi}$$

$$\nabla \times A = \begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

(31)

## Short dipole antenna

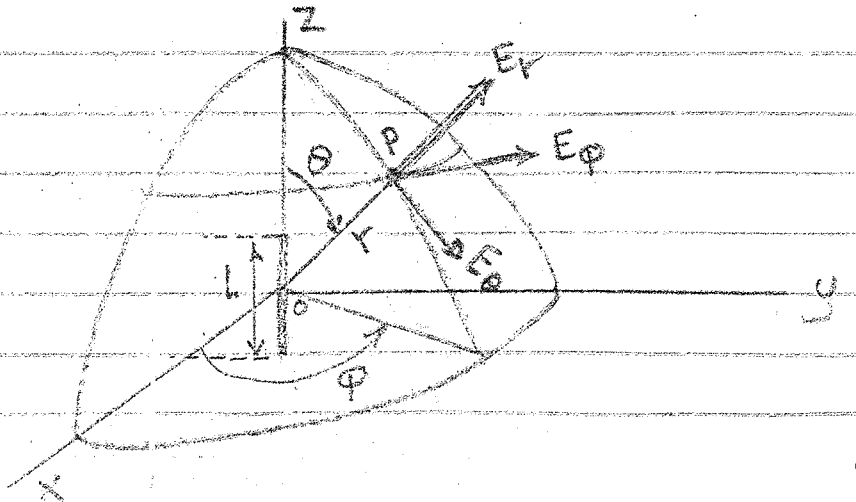
A short linear conductor is often called a short dipole. The length " $L$ " is very short compared with the wavelength ( $L \ll \lambda$ ). The diameter " $d$ " of the dipole is small compared with its length ( $d \ll L$ ).

~~The current is uniform (I) along the entire length L of the dipole (This dipole called Hertzian dipole or current element).~~  
 The current is uniform ( $I$ ) along the entire length " $L$ " of the dipole (This dipole called Hertzian dipole or current element). For purposes of analysis we may consider that the short dipole consists simply of a thin conductor of length " $L$ " with a uniform current " $I$ " and point charges " $q$ " at the ends.

$$\frac{\partial q}{\partial t} = I$$



Let us now proceed to find the fields everywhere around a short dipole. Let the dipole of length " $L$ " be placed coincident with the  $z$  axis and with its center at the origin.



The electric field intensity  $E$  at any point  $P$  is expressed by

$$E = -\nabla V - \frac{\partial A}{\partial t} \quad [V/m]$$

where  $V$  - electric scalar potential at point  $P$ ,  $[V]$

$A$  - vector potential at point  $P$ ,  $Wb/m$

The magnetic field  $H$  at any point  $P$  is

$$H = \frac{1}{\mu_0} \nabla \times A \quad (A/m)$$

where  $\mu_0$  - permeability of air ( $400\pi$  <sup>nano</sup>  $nH/m$ )

In far-field we must use the retarded potentials.

$$E = -\nabla[V] - \frac{\partial[A]}{\partial t} = -\nabla[V] - j\omega[A] \quad (V/m)$$

$$H = \frac{1}{\mu_0} \nabla \times [A] \quad [A/m]$$

$[V], [A]$  - retarded

$$[V] = \frac{1}{4\pi\epsilon_0} \int_V \frac{\rho_0 e^{j\omega[t-(r/c)]}}{r} d\tau, [V]$$

$$[A] = \frac{\mu_0}{4\pi} \int_V \frac{J_0 e^{j\omega[t-(r/c)]}}{r} d\tau \quad [Wb/m], J_0 \left[ \frac{A}{m^2} \right]$$

From Last figure the current is entirely in the  $Z$  direction. Hence, it follows that the retarded vector potential has only a  $Z$  component.

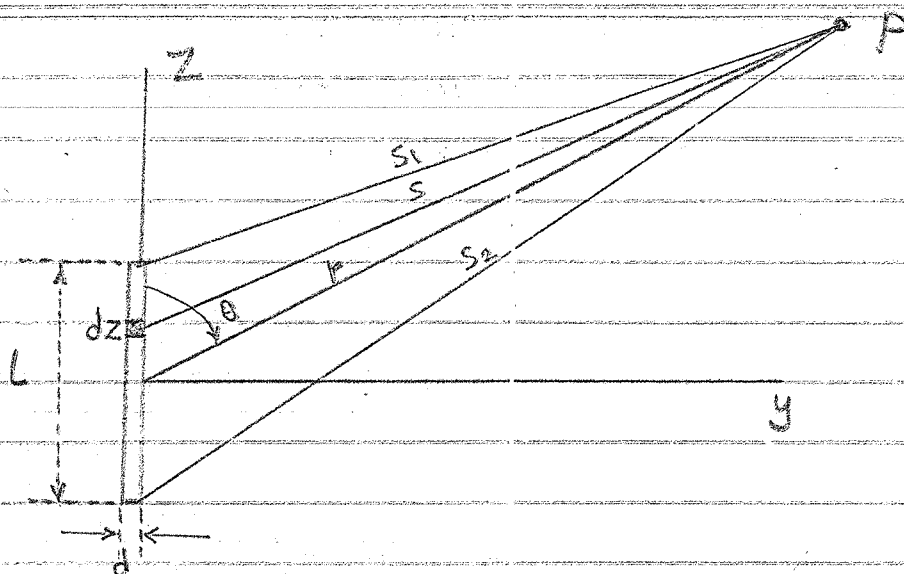
$$A_z = A = \frac{\mu_0 I_0}{4\pi} \int_{-L/2}^{L/2} \frac{e^{j\omega[t-\beta s]}}{s} dz$$

$$\beta s = \frac{2\pi}{\lambda} s = \beta r = \frac{2\pi}{\lambda} r$$

$$\omega r/c = \frac{2\pi}{T} r/c = \frac{2\pi}{T} \frac{r}{c} = \frac{2\pi}{\lambda} r$$

$$\omega s/c = \frac{2\pi}{T} \frac{s}{c} = \frac{2\pi}{\lambda} s$$

33



$d \rightarrow 0$

$I_0$  - amplitude (peak value in time) of current (same at all points along dipole), A

Since  $r \gg L$  we can put  $r = s$

Since  $\lambda \gg L$  we can neglect the magnitude and phase differences of the contribution from the different parts of the wire. Thus last equation becomes

$$A_z = \frac{\mu_0 I_0 L e^{j(\omega t - \beta r)}}{4\pi r}$$

$$V = \frac{q_0}{4\pi\epsilon_0} \left( \frac{e^{j(\omega t - \beta s_1)}}{s_1} - \frac{e^{j(\omega t - \beta s_2)}}{s_2} \right), [V]$$

where  $q_0$  - amplitude (peak value in time) of charge at ends of dipole, [Coulombs]

$$q = q_0 e^{j(\omega t - \beta s)} - \text{retarded charge, [Coulomb]}$$

$$I = I_0 e^{j(\omega t - \beta s)} - \text{retarded current [A]}$$

$$q = \int I dt$$

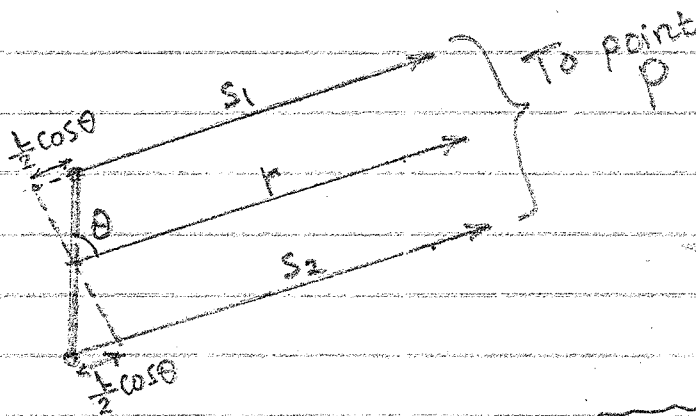
34

$$q_0 e^{j(\omega t - \beta s)} = \int I_0 e^{j(\omega t - \beta s)} dt = \int I_0 e^{j(\omega t - \beta s)} \frac{d(j\omega t - \beta s)}{j\omega}$$

$$q_0 = \frac{I_0}{j\omega}$$

$$V = \frac{I_0}{4\pi\epsilon_0 j\omega} \left( \frac{e^{j[\omega t - \beta s_1]}}{s_1} - \frac{e^{j[\omega t - \beta s_2]}}{s_2} \right)$$

When  $r \gg L$ , the lines of Length  $s_1$  and  $s_2$  from the ends of the dipole to the point  $P$  may be considered parallel, as shown in figure:



$$s_1 = r - \frac{L}{2} \cos \theta$$

$$s_2 = r + \frac{L}{2} \cos \theta$$

$$V = \frac{I_0}{4\pi\epsilon_0 j\omega} \left[ \frac{e^{j\omega(t - \frac{r - \frac{L}{2} \cos \theta}{c})}}{r - \frac{L}{2} \cos \theta} - \frac{e^{j\omega(t - \frac{r + \frac{L}{2} \cos \theta}{c})}}{r + \frac{L}{2} \cos \theta} \right]$$

$$= \frac{I_0 e^{j\omega(t - \frac{r}{c})}}{j\omega 4\pi\epsilon_0} \left[ \frac{e^{\frac{j\omega L \cos \theta}{2c}}}{r - \frac{L}{2} \cos \theta} - \frac{e^{\frac{j\omega L \cos \theta}{2c}}}{r + \frac{L}{2} \cos \theta} \right]$$

$$\beta s_1 = \beta(r - \frac{L}{2} \cos \theta)$$

$$= \frac{2\pi}{\lambda} r - \frac{2\pi}{\lambda} \frac{L}{2} \cos \theta$$

$$= \frac{2\pi}{\lambda} r - \frac{2\pi}{\lambda} \frac{L}{2} \cos \theta$$

$$= \frac{\omega}{c} r - \frac{\omega}{c} \frac{L}{2} \cos \theta$$

$$= \omega \left( \frac{r - \frac{L}{2} \cos \theta}{c} \right)$$



(35)

$$\text{Let } X = \frac{\omega L \cos \theta}{2 \omega} = \frac{2 \pi L \cos \theta}{2 \pi \omega} = \frac{2 \pi L \cos \theta}{2 \lambda} = \frac{\pi L \cos \theta}{\lambda}$$

$$\therefore \frac{L}{\lambda} < 1$$

$$\therefore X < 1 \Rightarrow e^{jX} = 1 + jX ; e^{-jX} = 1 - jX$$

$$V = \frac{I_0 e^{j\omega(t - \frac{r}{\omega})}}{j\omega 4\pi \epsilon_0} \left[ \frac{1 + j \frac{\omega L \cos \theta}{2 \omega}}{r - \frac{L}{2} \cos \theta} - \frac{1 - j \frac{\omega L \cos \theta}{2 \omega}}{r + \frac{L}{2} \cos \theta} \right]$$

$$= \frac{I_0 e^{j\omega(t - \frac{r}{\omega})}}{j\omega 4\pi \epsilon_0} \left[ \frac{(1 + j \frac{\omega L \cos \theta}{2 \omega})(r + \frac{L}{2} \cos \theta) - (1 - j \frac{\omega L \cos \theta}{2 \omega})(r - \frac{L}{2} \cos \theta)}{r^2 - \frac{L^2}{4} \cos^2 \theta} \right]$$

$$r^2 - \frac{L^2}{4} \cos^2 \theta \approx r^2 \quad (L < r)$$

$$V = \frac{I_0 e^{j\omega(t - \frac{r}{\omega})}}{j\omega 4\pi \epsilon_0} \left[ \frac{j \frac{\omega L \cos \theta}{\omega} + L \cos \theta}{r^2} \right] = \frac{I_0 e^{j\omega(t - \frac{r}{\omega})}}{j\omega 4\pi \epsilon_0} \left[ j \frac{\omega L \cos \theta}{\omega r} + \frac{L \cos \theta}{r^2} \right]$$

$$= \frac{I_0 e^{j\omega(t - \frac{r}{\omega})}}{4\pi \epsilon_0} \left[ \frac{L \cos \theta}{\omega r} + \frac{L \cos \theta}{j\omega r^2} \right] = \frac{I_0 L \cos \theta e^{j\omega(t - \frac{r}{\omega})}}{4\pi \epsilon_0} \left[ \frac{1}{\omega r} + \frac{1}{j\omega r^2} \right]$$

$$V = \frac{I_0 L \cos \theta e^{j\omega(t - \frac{r}{\omega})}}{4\pi \epsilon_0} \left[ \frac{1}{\omega r} + \frac{1}{j\omega r^2} \right]$$

Electric and magnetic fields are obtained by the equations:

$$E = -\frac{dA}{dt} - \nabla V, \quad V - \text{electric scalar potential}$$

A - magnetic vector potential

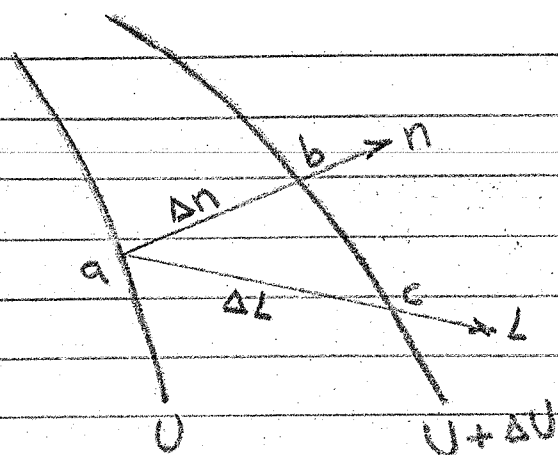
$$H = \frac{1}{\mu} (\nabla \times A)$$

$\nabla V = \text{grad } V = \text{gradient of } V$

$\nabla \times A = \text{Curl } A$

Gradient of a scalar field  $V$  is a vector that represents both the magnitude and the direction of the maximum space rate of increase of  $V$ .

(36)



$$\text{grad } U = \frac{dU}{dn} \hat{n}$$

$$\text{grad}_L U = \frac{dU}{dn} \cos(L, n) = \frac{dU}{dL}$$

$\mu$ -permeability  $\left[ \frac{\text{henry}}{\text{meter}} \right]$   
 $\epsilon$ -permittivity  $\left[ \frac{\text{farad}}{\text{meter}} \right]$

$$\text{grad}_x U = \frac{dU}{dn} \cos(n, x) = \frac{dU}{dx}$$

$$\text{grad}_y U = \frac{dU}{dn} \cos(n, y) = \frac{dU}{dy}$$

$$\text{grad}_z U = \frac{dU}{dn} \cos(n, z) = \frac{dU}{dz}$$

$$|\text{grad } U| = \frac{dU}{dn} = \sqrt{\left(\frac{dU}{dx}\right)^2 + \left(\frac{dU}{dy}\right)^2 + \left(\frac{dU}{dz}\right)^2}$$

$$dU = -E \, dn$$

$$E = -\text{grad } U$$

$$\nabla = \frac{d}{dx} \mathbf{a}_x + \frac{d}{dy} \mathbf{a}_y + \frac{d}{dz} \mathbf{a}_z$$

$$\text{grad } U = \nabla U = \frac{dU}{dx} \mathbf{a}_x + \frac{dU}{dy} \mathbf{a}_y + \frac{dU}{dz} \mathbf{a}_z$$

curl of a vector

curl of  $A$  as an axial (or rotational) vector whose magnitude is the maximum circulation of  $A$  per unit area as the area tends to zero and whose direction is the normal direction of the

maximum. That is

$$\text{Curl } A = \nabla \times A = \left( \lim_{\Delta S \rightarrow 0} \frac{\oint_L A \cdot dL}{\Delta S} \right) a_n$$

Where the area  $\Delta S$  is bounded by the curve  $L$  and  $a_n$  is the unit vector normal to the surface  $\Delta S$  and is determined using the right hand rule.

$$\nabla \times A = \left[ \frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right] a_x + \left[ \frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} \right] a_y + \left[ \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right] a_z$$

$$\nabla \times A = \begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} \quad - \text{ [Cartesian coordinates]}$$

In spherical coordinates :

$$\text{Curl } A = \nabla \times A = \frac{1}{r^2 \sin \theta} \begin{vmatrix} a_r & r a_\theta & r \sin \theta a_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_r & r A_\theta & r \sin \theta A_\phi \end{vmatrix}$$

$$= \frac{1}{r \sin \theta} \left[ \frac{\partial (A_\phi \sin \theta)}{\partial \theta} - \frac{\partial A_\theta}{\partial \phi} \right] a_r + \frac{1}{r} \left[ \frac{1}{\sin \theta} \frac{\partial A_r}{\partial \phi} - \frac{\partial (r A_\phi)}{\partial r} \right] a_\theta$$

$$+ \frac{1}{r} \left[ \frac{\partial (r A_\theta)}{\partial r} - \frac{\partial A_r}{\partial \theta} \right] a_\phi$$

$$\text{grad } U = \nabla U = \frac{\partial U}{\partial r} a_r + \frac{\partial U}{\partial \theta} \frac{a_\theta}{r} + \frac{\partial U}{\partial \phi} \frac{a_\phi}{r \sin \theta}$$

38

Substituting in equation  $[E = -j\omega A - \nabla V]$  and equating the corresponding terms.

$$E_r = -j\omega A_r - \frac{\partial V}{\partial r}$$

$$E_\theta = -j\omega A_\theta - \frac{1}{r} \frac{\partial V}{\partial \theta}$$

$$E_\phi = -j\omega A_\phi - \frac{1}{r \sin \theta} \frac{\partial V}{\partial \phi}$$

$$A_z = \frac{\mu}{4\pi} \frac{I_0 e^{-j\omega(t - \frac{r}{c})} L}{r}$$

$$A_r = \frac{\mu}{4\pi} \frac{I_0 e^{-j\omega(t - \frac{r}{c})} L}{r} \cos \theta$$

$$\frac{dA}{dt} = d(A e^{j\omega t}) / dt = j\omega A e^{j\omega t}$$

$$H = \frac{1}{\mu} \text{Curl } A$$

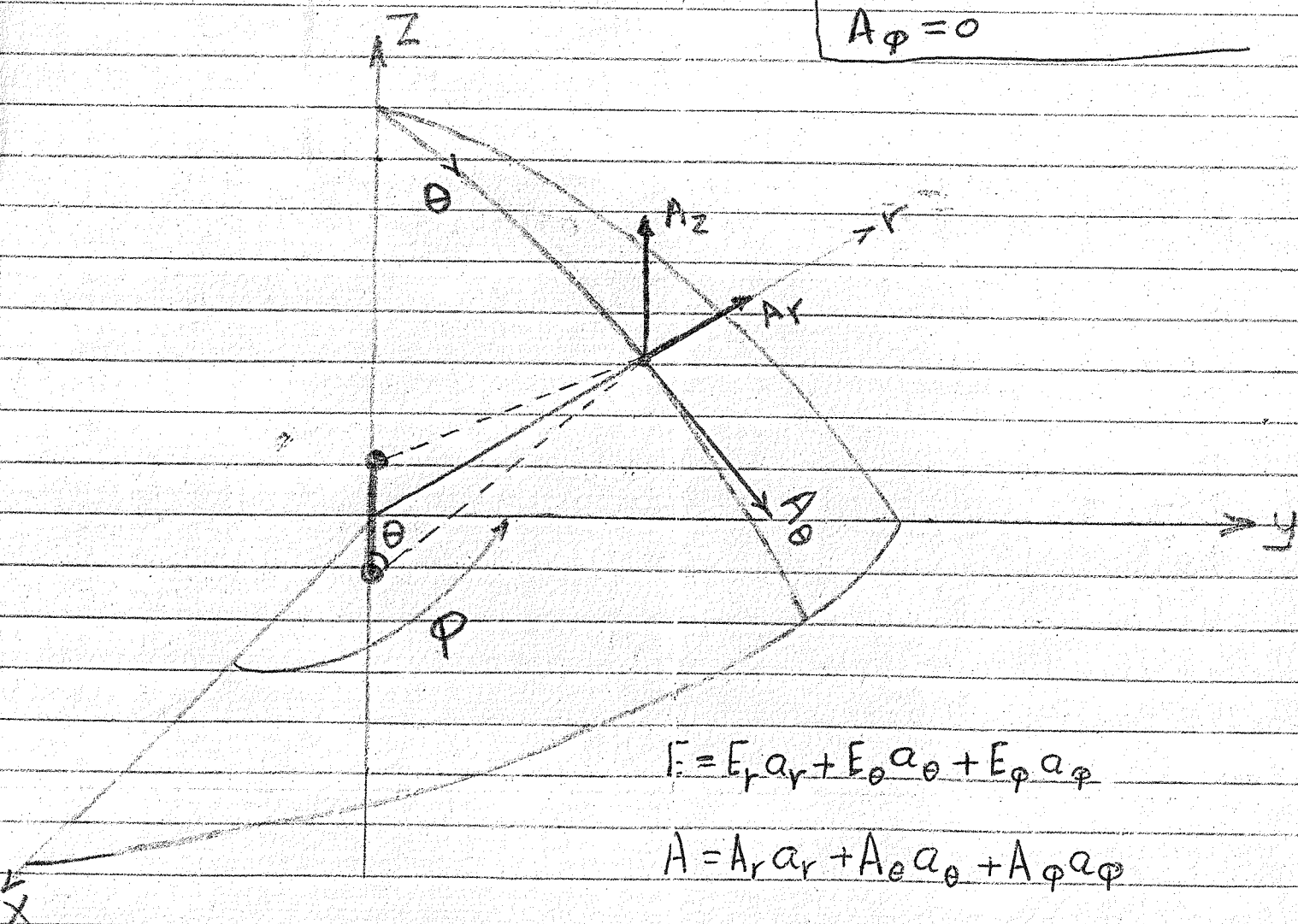
$L$  - very small

$A = A_z$  - the vector potential exists only along  $z$

$A_r = A_z \cos \theta$  - in polar coordinate

$A_\theta = -A_z \sin \theta$

$A_\phi = 0$



(39)

$$E_r = -j\omega A_r - \frac{\partial V}{\partial r}$$

$$= -j\omega \frac{\mu I_0 L \cos\theta}{4\pi r} e^{j\omega(t - \frac{r}{u})} - \frac{\partial}{\partial r} \left[ \frac{I_0 L \cos\theta}{4\pi\epsilon} \left[ \frac{e^{j\omega(t - \frac{r}{u})}}{ur} + \frac{e^{j\omega(t - \frac{r}{u})}}{j\omega r^2} \right] \right]$$

$$= -j\omega \frac{\mu I_0 L \cos\theta}{4\pi r} e^{j\omega(t - \frac{r}{u})} - \frac{I_0 L \cos\theta}{4\pi\epsilon} \left[ \frac{j\omega}{u^2 r^2} e^{j\omega(t - \frac{r}{u})} - \frac{j\omega}{u} e^{j\omega(t - \frac{r}{u})} + \frac{j\omega r^2}{(j\omega r^2)^2} e^{j\omega(t - \frac{r}{u})} - j2\omega r e^{j\omega(t - \frac{r}{u})} \right]$$

$$= \frac{I_0 L \cos\theta}{4\pi\epsilon} e^{j\omega(t - \frac{r}{u})} \left[ \frac{-j\omega\mu\epsilon}{r} + \frac{j\omega}{u^2 r} + \frac{1}{ur^2} + \frac{1}{ur^2} + \frac{2}{j\omega r^3} \right], \quad \left[ \mu\epsilon = \frac{1}{u^2} \right]$$

$$= \frac{I_0 L \cos\theta}{4\pi\epsilon} e^{j\omega(t - \frac{r}{u})} \left[ \frac{-j\omega}{r u^2} + \frac{j\omega}{r u^2} + \frac{2}{ur^2} + \frac{2}{j\omega r^3} \right] \quad \left[ u = c = 3 \times 10^8 \text{ m/s} \right]$$

$$= \frac{I_0 L \cos\theta}{2\pi\epsilon} e^{j\omega(t - \frac{r}{u})} \left[ \frac{1}{ur^2} + \frac{1}{j\omega r^3} \right]$$

$$E_\theta = -j\omega A_\theta - \frac{1}{r} \frac{\partial V}{\partial \theta}, \quad A_\theta = -A_z \sin\theta$$

$$= -j\omega \left[ \frac{-\mu I_0 L \sin\theta}{4\pi r} e^{j\omega(t - \frac{r}{u})} \right] - \frac{1}{r} \frac{\partial}{\partial \theta} \left[ \frac{I_0 L \cos\theta}{4\pi\epsilon} e^{j\omega(t - \frac{r}{u})} \left( \frac{1}{r u} + \frac{1}{j\omega r^2} \right) \right]$$

$$= j\omega \left[ \frac{\mu I_0 L \sin\theta}{4\pi r} e^{j\omega(t - \frac{r}{u})} \right] + \frac{1}{r} \frac{I_0 L}{4\pi\epsilon} e^{j\omega(t - \frac{r}{u})} \sin\theta \left( \frac{1}{r u} + \frac{1}{j\omega r^2} \right)$$

$$= \frac{I_0 L \sin\theta}{4\pi\epsilon} e^{j\omega(t - \frac{r}{u})} \left[ \frac{j\omega\mu\epsilon}{r} + \frac{1}{j\omega r^3} + \frac{1}{ur^2} \right]$$

$$E_\theta = \frac{I_0 L \sin\theta}{4\pi\epsilon} e^{j\omega(t - \frac{r}{u})} \left[ \frac{j\omega}{r u^2} + \frac{1}{j\omega r^3} + \frac{1}{ur^2} \right]$$

$$E_\phi = 0 \quad (A_\phi = 0)$$

$$B = \nabla \times A = \text{curl } A, \quad B = \mu H$$

$$H = \frac{1}{\mu} \nabla \times A = \frac{1}{\mu} \text{curl } A$$

In spherical coordinates:

$$H = H_r a_r + H_\theta a_\theta + H_\phi a_\phi$$

40

$$\text{curl}_r A = \frac{1}{r \sin \theta} \left[ \frac{\partial}{\partial \theta} (\sin \theta \cdot A_\phi) - \frac{\partial A_\theta}{\partial \phi} \right]$$

$$\text{curl}_\theta A = \frac{1}{r \sin \theta} \left[ \frac{\partial A_r}{\partial \phi} - \frac{\partial}{\partial r} (r \sin \theta \cdot A_\phi) \right]$$

$$\text{curl}_\phi A = \frac{1}{r} \left[ \frac{\partial}{\partial r} (r \cdot A_\theta) - \frac{\partial A_r}{\partial \theta} \right]$$

$$H_\phi = \frac{1}{\mu r} \left[ \frac{\partial}{\partial r} \left( -r \frac{\mu I_0 e^{j\omega(t-\frac{r}{c})}}{4\pi r} \sin \theta \right) - \frac{\partial}{\partial \theta} \left( \frac{\mu I_0 e^{j\omega(t-\frac{r}{c})}}{4\pi r} \cos \theta \right) \right]$$

$$= \frac{I_0 e^{j\omega(t-\frac{r}{c})}}{4\pi} \sin \theta \left[ \frac{j\omega}{r^2} + \frac{1}{r^2} \right]$$

$$E_r = \frac{I_0 e^{j\omega(t-\frac{r}{c})}}{2\pi \epsilon} \cos \theta \left[ \frac{1}{j\omega r^3} + \frac{1}{\omega^2 r^2} \right]$$

$$E_\theta = \frac{I_0 e^{j\omega(t-\frac{r}{c})}}{4\pi \epsilon} \sin \theta \left[ \frac{j\omega}{\omega^2 r} + \frac{1}{j\omega r^3} + \frac{1}{\omega^2 r^2} \right]$$

$$E_\phi = 0$$

$$H_r = 0$$

$$H_\theta = 0$$

$$H_\phi = \frac{I_0 e^{j\omega(t-\frac{r}{c})}}{4\pi} \sin \theta \left[ \frac{j\omega}{r^2} + \frac{1}{r^2} \right]$$

When  $\beta r \gg 1$  or  $2\pi r \gg \lambda$  the  $\frac{1}{r^3}$  and  $\frac{1}{r^2}$  terms can be neglected in favor of the  $\frac{1}{r}$  term.

Region of  $\beta r \gg 1$  [ $r \gg \lambda$ ] called Far-field (Radiation field)

In near field (induction field) term  $\frac{1}{r^3}$  called electrostatic



41

In far field :

$$E_{\phi} = 0, H_r = 0$$

$$E_r = 0, H_{\theta} = 0$$

$$j\omega \frac{r}{u} = \frac{j2\pi}{T} r = j\beta r; T u = \lambda$$

$$\frac{\omega}{4\pi\epsilon u^2} = \frac{2\pi u}{\lambda \cdot 4\pi\epsilon u^2} = \frac{1}{2\lambda\epsilon u} = \frac{\sqrt{\mu\epsilon}}{2\lambda}$$

$$= \frac{1}{2\lambda} \sqrt{\frac{\mu}{\epsilon}} = \frac{120\pi}{2\lambda} = \frac{60\pi}{\lambda}$$

$$\eta = \sqrt{\frac{\mu}{\epsilon}} = 120\pi$$

$$E_{\theta} = \frac{I_0 e^{j\omega(t - \frac{r}{u})}}{4\pi\epsilon} L \sin\theta \left[ \frac{j\omega}{u^2 r} \right] = \frac{j60\pi I_0 e^{j\omega(t - \frac{r}{u})}}{r\lambda} L \sin\theta$$

$$E_{\theta} = \frac{j30\beta I_0 e^{j\omega t}}{r} L \sin\theta e^{-j\beta r} \quad (*)$$

$$H_{\phi} = \frac{j I_0 e^{j\omega t}}{2r\lambda} L \sin\theta e^{-j\beta r} = \frac{j\beta}{4\pi r} I_0 e^{j\omega t} L \sin\theta e^{-j\beta r}$$

$$= \frac{j2\pi I_0 L e^{j\omega t} \sin\theta}{4\lambda\pi r} \cdot \frac{u}{u} e^{-j\beta r}, \quad \beta = \frac{2\pi}{\lambda}$$

$$= j \frac{2\pi f I_0 L \sin\theta}{4\pi r u} e^{-j\beta r} e^{j\omega t}$$

$$H_{\phi} = \frac{j\omega I_0 e^{j\omega t}}{4\pi r u} L \sin\theta e^{-j\beta r}$$

$$= \frac{j \frac{2\pi}{T} I_0 L \sin\theta e^{j\omega t} e^{-j\beta r}}{4\pi r}$$

$$= \frac{j\beta I_0 L \sin\theta}{4\pi r} e^{j[\omega t - \beta r]}$$

$$H_{\phi} = \frac{j\beta I_0 L \sin\theta}{4\pi r} e^{j[\omega t - \beta r]}$$

$$\omega t - \beta r = \omega t - \frac{2\pi}{\lambda} r$$

$$= \omega t - \frac{2\pi}{\lambda} r$$

$$= \omega t - \frac{\omega}{u} r$$

$$= \omega(t - \frac{r}{u})$$

$$\eta = \frac{E_{\theta}}{H_{\phi}} = 120\pi$$

42

The total power radiated by current element (hertzian dipole) can be computed by integrating the radial Poynting vector over a spherical surface centered at the element. The element of area on the spherical shell will be taken as the strip  $da$  where

$$da = 2\pi r \sin\theta \cdot r d\theta = 2\pi r^2 \sin\theta d\theta$$

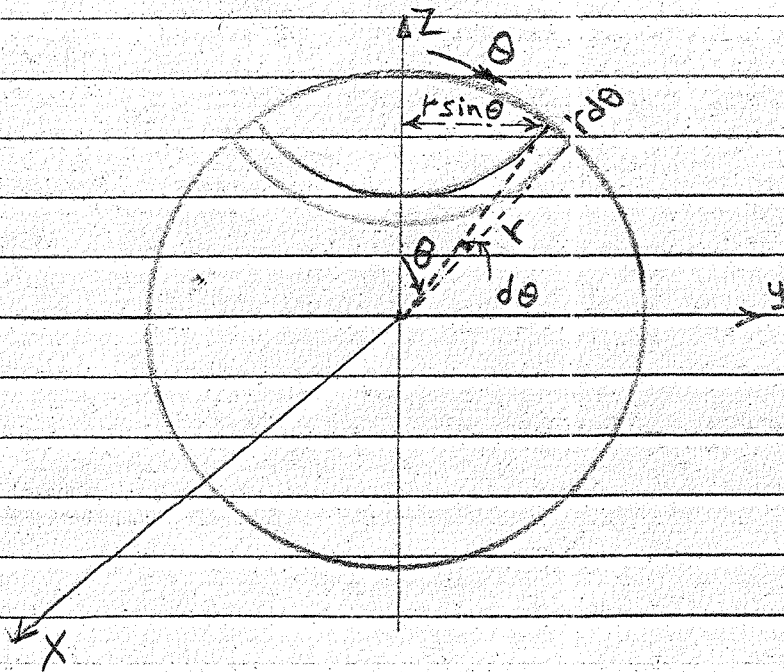
$$P_r = \oint_S \frac{H^2}{2} da = \oint_S \frac{1}{2} \left( \frac{\omega I_0 L \sin\theta}{4\pi r} \right)^2 2\pi r^2 \sin\theta d\theta$$

because  $H$  is peak (amplitude) value

$$= \frac{\omega^2 I_0^2 L^2}{16\pi} \int_0^\pi \sin^3\theta d\theta = \frac{\omega^2 I_0^2 L^2}{12\pi} = \frac{120\pi \cdot (2\pi/\lambda)^2 I_0^2 L^2}{12\pi} =$$

$$= \frac{40\pi^2 I_0^2 L^2}{\lambda^2} = \frac{40\pi^2 I_0^2 L^2}{\lambda^2} = 40\pi^2 \left(\frac{L}{\lambda}\right)^2 I_0^2$$

$$P_r = \frac{I_0^2 R_r}{2}, \quad R_r = \frac{2P_r}{I_0^2} = 80\pi^2 \left(\frac{L}{\lambda}\right)^2 - \text{radiation resistance}$$



if the distribution of current is not uniform for short dipole, we find the average current

$$I_0 \text{ --- } L \text{ --- } I_{AV}$$

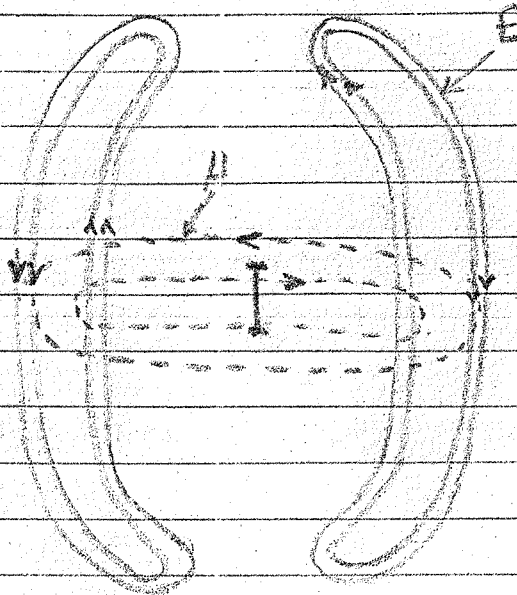
$$\frac{L I_0}{2} = I_{AV} L$$

$$I_{AV} = \frac{1}{2} I_0$$

$$R_r = 80\pi^2 \left(\frac{L}{\lambda}\right)^2 \left(\frac{I_{AV}}{I_0}\right)^2 = 80\pi^2 \left(\frac{L}{\lambda}\right)^2 \cdot \frac{1}{4}$$

43

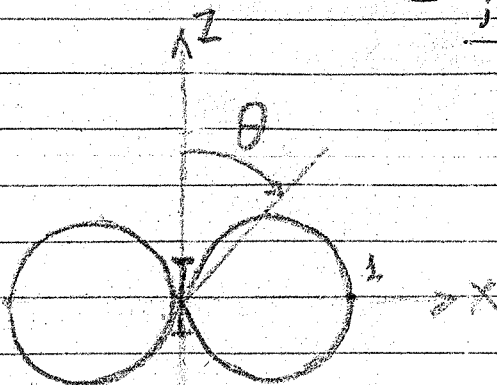
Field structure in far field of current element is show in figure below:



The field pattern of the Hertzian (current element) dipole is obtained from eq. (\*)

The normalized  $f(\theta) = \frac{E_\theta}{E_{\theta \max}}$  [E-plane pattern]

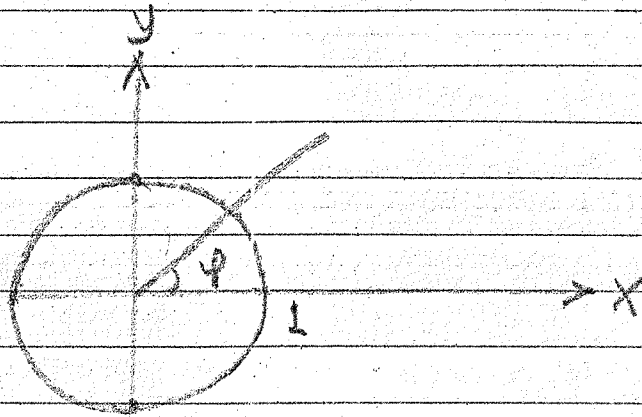
$$= \frac{j30\beta I_0 L \sin\theta}{r} \div \frac{j30\beta I_0 L}{r} = \sin\theta$$



(44)

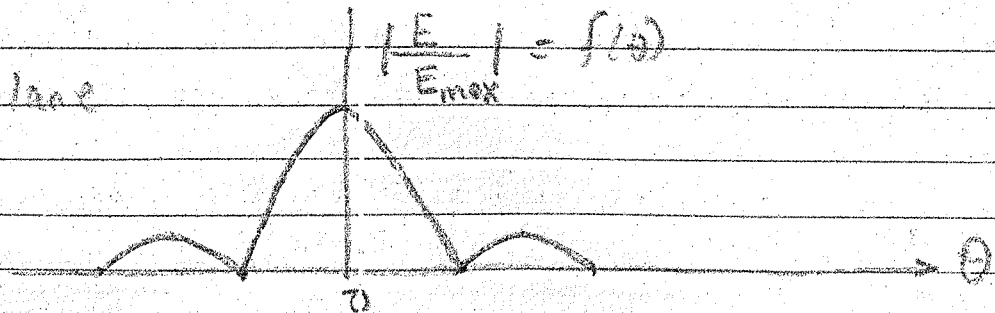
For H-plane pattern we set  $\theta = 90^\circ$  so that  $f(\theta) = 1$  which is circle of radius 1 as show in following figure

polar coordinate  
H-plane

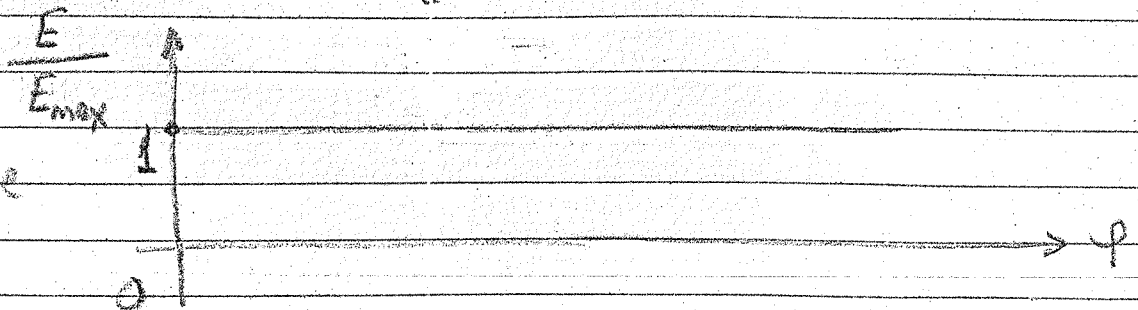


In Cartesian Coordinate :

a. E plane



b. H plane



(44)

Example

Calculate the radiation resistance of current element whose overall length is  $\lambda/50$ .

Solution

$$R_r = 80 \pi^2 \left( \frac{L}{\lambda} \right)^2 = 80 \pi^2 \left( \frac{\lambda}{50 \times \lambda} \right)^2 = 0.315507, \Omega$$

Example

Define radiation resistance. Calculate the radiation resistance and efficiency of a current element whose overall length is  $\frac{\lambda}{50}$  and loss resistance is  $1.5 \Omega$ .

Solution

Radiation resistance is that fictitious resistance which when connected in series with antenna, will consume the same amount of power as when actually radiating.

$$R_r = 80 \pi^2 \left( \frac{L}{\lambda} \right)^2 = 80 \pi^2 \left( \frac{\lambda}{50 \lambda} \right)^2 = 0.3158 \Omega$$

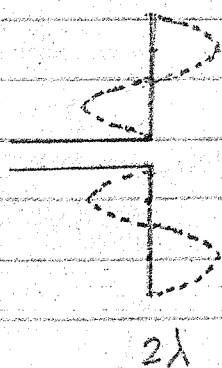
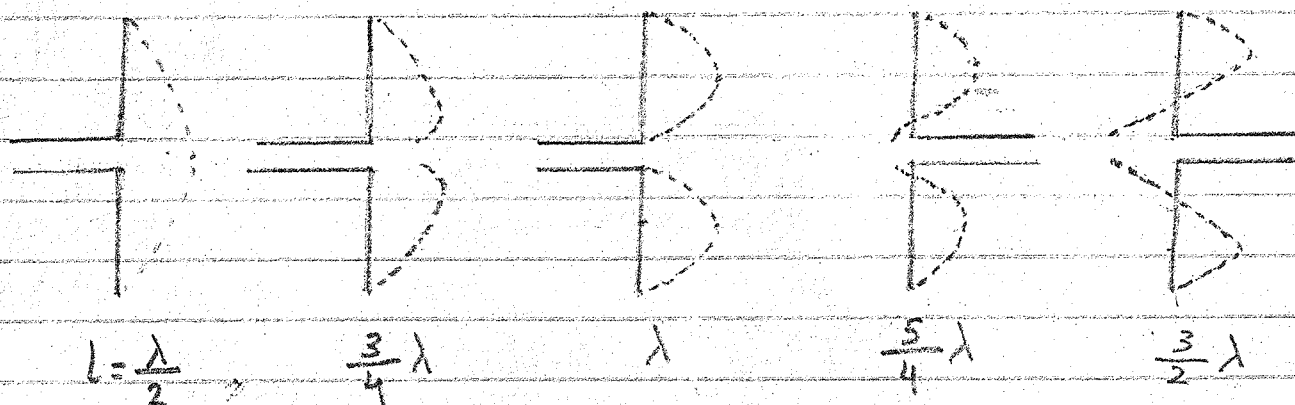
$$\eta = \text{efficiency} = \frac{R_r}{R_r + R_L} = \frac{0.3158}{0.3158 + 1.5} = 0.1739$$

$$\eta \% = 17.4 \%$$

## The thin Linear antenna

The analysis will be assumed for the far-field patterns of thin linear antenna. It is assumed that the antennas are symmetrically fed at the center by a balanced two-wire transmission line. The antennas may be of any length, but it is assumed that the current distribution is sinusoidal. The antenna is thin, i.e. when the conductor diameter is less than, say,  $\frac{\lambda}{100}$ .

Examples of the approximate current distributions on a number of thin linear center-fed antennas of different length are illustrated in following figure.



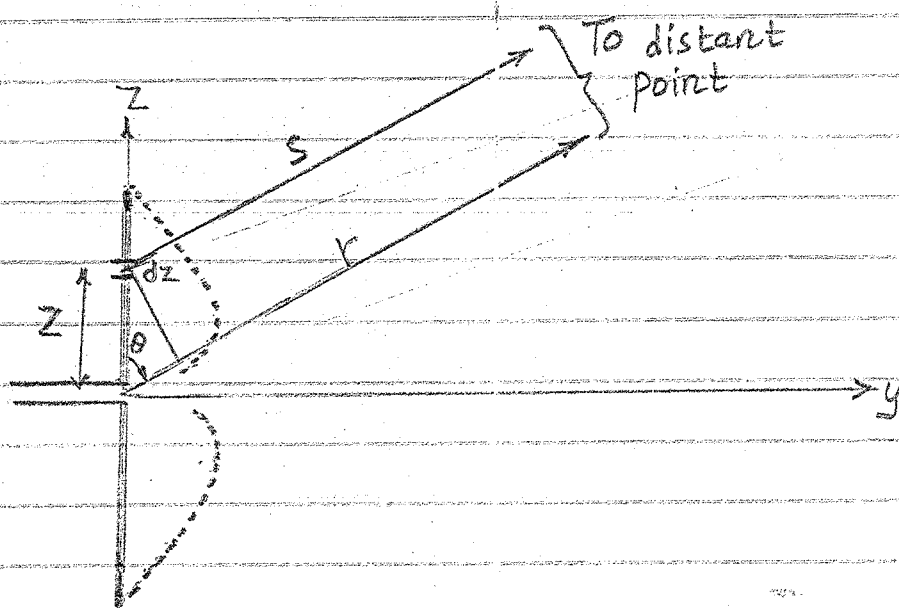
The currents are in phase over each  $\lambda/2$  section and in opposite phase over the next.



48

Let us now proceed to develop the far-field equations for a symmetrical, thin, linear, center-fed antenna referred to a point at a distance  $S$  is

$$[I] = I_0 \sin \left[ \frac{2\pi}{\lambda} \left( -\frac{L}{2} \pm z \right) \right] e^{j\omega[t - (r/v)]}$$



By regarding the antenna as made up of a series of infinitesimal dipoles of length  $dz$ , the field of the entire antenna may then be obtained by integrating the fields from all of the dipoles making up the antenna.

The far-fields  $dE_\theta$  and  $dH_\phi$  at a distance  $S$  from the infinitesimal dipole  $dz$  are

$$dE_\theta = \frac{j60\pi [I] \sin\theta dz}{S\lambda} = \frac{j30\beta [I] \sin\theta dz}{S}$$

$$dH_\phi = \frac{j[I] \sin\theta dz}{2S\lambda}$$

Since  $E_\theta = 120\pi H_\phi$ , it will suffice to calculate  $H_\phi$ .

$$dE_\theta = 120\pi dH_\phi$$

(42)

$$H_{\phi} = \int_{-L/2}^{L/2} dH_{\phi}$$

$H_{\phi}$  - magnetic field of entire antenna.

$$H_{\phi} = \frac{jI_0 \sin \theta e^{j\omega t}}{2\lambda} \left\{ \int_{-L/2}^0 \frac{1}{s} \sin \left[ \frac{2\pi}{\lambda} \left( \frac{L}{2} + z \right) \right] e^{-j\omega s/r} dz + \int_0^{L/2} \frac{1}{s} \sin \left[ \frac{2\pi}{\lambda} \left( \frac{L}{2} - z \right) \right] e^{-j\omega s/r} dz \right\} \quad (*)$$

At a large distance, the difference between  $s$  and  $r$  can be neglected in its effect on the amplitude but its effect on the phase must be considered

$$s = r - z \cos \theta$$

Substituting last equation into equation (\*) and also  $r$  for  $s$  in the amplitude factor, (\*) becomes

$$H_{\phi} = \frac{jI_0 \sin \theta e^{j\omega [t - (\frac{r}{c})]}}{2\lambda r} \left\{ \int_{-L/2}^0 \sin \left[ \frac{2\pi}{\lambda} \left( \frac{L}{2} + z \right) \right] e^{j(\omega \cos \theta)z/c} dz + \int_0^{L/2} \sin \left[ \frac{2\pi}{\lambda} \left( \frac{L}{2} - z \right) \right] e^{j(\omega \cos \theta)z/c} dz \right\}$$

Since  $\beta = \frac{\omega}{c} = \frac{2\pi}{\lambda}$  and  $\frac{\beta}{4\pi} = \frac{1}{2\lambda}$ , last equation may be

rewritten as

$$H_{\phi} = \frac{j\beta I_0 \sin \theta e^{j\omega [t - (\frac{r}{c})]}}{4\pi r} \left\{ \int_{-L/2}^0 e^{j\beta z \cos \theta} \sin \left[ \beta \left( \frac{L}{2} + z \right) \right] dz + \int_0^{L/2} e^{j\beta z \cos \theta} \sin \left[ \beta \left( \frac{L}{2} - z \right) \right] dz \right\}$$

48

The integrals are of the form

$$\int e^{ax} \sin(c+bx) dx = \frac{e^{ax}}{a^2+b^2} [a \sin(c+bx) - b \cos(c+bx)]$$

where for the first integral

$$a = j\beta \cos\theta$$

$$c = \beta L/2$$

$$b = \beta$$

for the second integral

$$a = j\beta \cos\theta$$

$$c = \beta L/2$$

$$b = -\beta$$

carrying through the two integrations, adding the results and simplifying yields

$$H_\phi = \frac{j[I_0]}{2\pi r} \left[ \frac{\cos[(\beta L \cos\theta)/2] - \cos(\beta L/2)}{\sin\theta} \right]$$

Multiplying  $H_\phi$  by  $Z = 120\pi$  gives  $E_\theta$  as

$$E_\theta = \frac{j60[I_0]}{r} \left[ \frac{\cos[(\beta L \cos\theta)/2] - \cos(\beta L/2)}{\sin\theta} \right]$$

where  $[I_0] = I_0 e^{j\omega(t - \frac{r}{c})}$

Last two equations are expressions for the far fields,  $H_\phi$  and  $E_\theta$ , of a symmetrical, center-fed, thin linear antenna of length  $L$ . The shape of the far-field pattern is

49

given by the factor in the brackets.

$$F = \frac{\cos[(\beta L \cos \theta)/2] - \cos(\beta L/2)}{\sin \theta}$$

The factors preceding the brackets in equation of  $E_\theta$  give the instantaneous magnitude of the fields as functions of the antenna current and the distance  $r$ .

As examples of the far-field patterns of linear center-fed antennas, three antennas of different lengths will be considered. Since the amplitude factor is independent of the length, only the relative field patterns as given by the pattern factor will be compared.

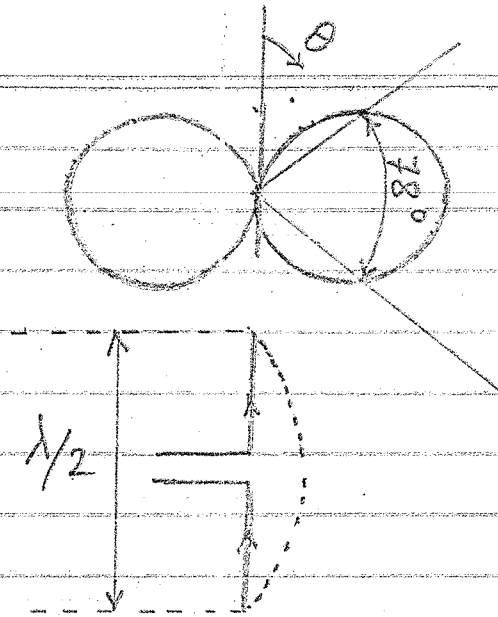
### Case. 1 $\lambda/2$ Antenna (half-wavelength dipole)

When  $L = \frac{\lambda}{2}$ , the pattern factor becomes

$$F = \frac{\cos[(\frac{\pi}{2}) \cos \theta]}{\sin \theta}, \quad \left[ \frac{2\pi}{\lambda} \frac{\lambda}{2} \cos \theta \right] / 2 = \frac{\pi \cos \theta}{2}$$

This pattern is shown in figure below. It is only slightly more directional than the pattern of an infinitesimal or short dipole which is given by  $\sin \theta$ . The beam width between half-power points of the  $\lambda/2$  antenna is  $78^\circ$  as compared to  $90^\circ$  for the short dipole.

50



## Case 2 Full-Wave antenna

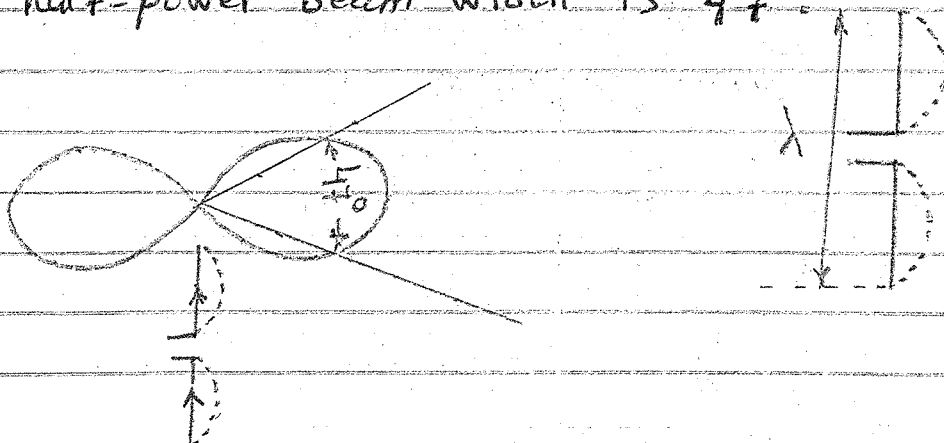
When  $L = \lambda$ , the pattern factor becomes

$$F(\theta) = \frac{\cos[\beta L \cos \theta / 2] - \cos(\frac{\beta L}{2})}{\sin \theta}$$

$$= \frac{\cos[\frac{2\pi}{\lambda} \lambda \cos \theta / 2] - \cos(\frac{2\pi \lambda}{2\lambda})}{\sin \theta}$$

$$F(\theta) = \frac{\cos(\pi \cos \theta) + 1}{\sin \theta}$$

The pattern for this case is shown in figure below  
The half-power beam width is  $47^\circ$ .



(5)

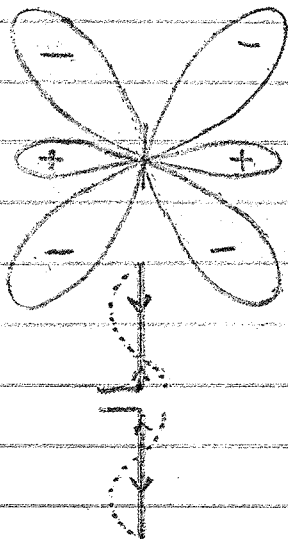
Case 3.  $3\lambda/2$  Antenna

When  $L = \frac{3\lambda}{2}$ , the pattern factor is

$$\begin{aligned}
 F(\theta) &= \frac{\cos[(\beta L \cos\theta)/2] - \cos[\beta L/2]}{\sin\theta} \\
 &= \frac{\cos\left[\left(\frac{2\pi}{\lambda} \frac{3\lambda}{2} \cos\theta\right)/2\right] - \cos\left[\frac{2\pi}{\lambda} \frac{3\lambda}{2} / 2\right]}{\sin\theta} \\
 &= \frac{\cos\left(\frac{3}{2}\pi \cos\theta\right) - \cos\left[-\frac{3}{2}\pi\right]}{\sin\theta}
 \end{aligned}$$

$$F(\theta) = \frac{\cos\left(\frac{3}{2}\pi \cos\theta\right)}{\sin\theta}$$

The pattern of this case is presented in figure below. With the midpoint of the antenna as phase center, the phase shift  $180^\circ$  at each null, the relative phase of the lobes being indicated by the + and - signs.





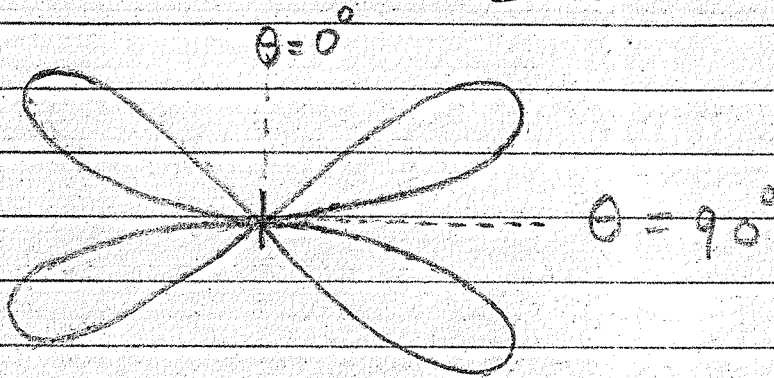
51.

Case 4.

$$L = 2\lambda$$

$$I(\theta) = \frac{\cos\left[\frac{2\pi}{\lambda} 2\lambda \cos\theta/2\right] - \cos\left[\frac{2\pi}{\lambda} 2\lambda/2\right]}{\sin\theta}$$

$$= \frac{\cos[2\pi \cos\theta] - \cos(2\pi)}{\sin\theta} = \frac{\cos[2\pi \cos\theta] - 1}{\sin\theta}$$



## 52 Radiation Resistance of Linear antenna

The total power radiated is given by

$$P_r = \iint S \cdot d\mathbf{s} = \frac{1}{2} \eta \iint |H_\phi|^2 d\mathbf{s}$$

$$= \frac{1}{2} \eta \int_0^{2\pi} \int_0^\pi |H_\phi|^2 r^2 \sin\theta d\theta d\phi, \quad \eta = \sqrt{\frac{\mu}{\epsilon}} = 120\pi$$

$$H_\phi = \frac{j[I_0]}{2\pi r} \left[ \frac{\cos[(\beta L \cos\theta)/2] - \cos(\beta L/2)}{\sin\theta} \right], \text{ where } [I_0] = I_0 e^{j\omega(t - \frac{r}{c})}$$

$I_0$  - peak value in time of the current

$$P_r = \frac{15 I_0^2}{\pi} \int_0^{2\pi} \int_0^\pi \frac{\{\cos[(\beta L/2) \cos\theta] - \cos(\beta L/2)\}^2}{\sin\theta} d\theta d\phi$$

Equating the radiated power as given by  $P_r = \frac{I_0^2 R_0}{2}$

$$\frac{I_0^2 R_0}{2} = 30 I_0^2 \int_0^\pi \frac{\{\cos[(\beta L/2) \cos\theta] - \cos(\beta L/2)\}^2}{\sin\theta} d\theta$$

$$R_0 = 60 \int_0^\pi \frac{\{\cos[(\beta L/2) \cos\theta] - \cos(\beta L/2)\}^2}{\sin\theta} d\theta, \text{ where the radiation}$$

resistance  $R_0$  is referred to the current maximum

After some extensive mathematical manipulations, it can be shown that last equation reduces to

$$R_0 = 60 \left\{ C + \ln(\beta L) - Ci(\beta L) + \frac{1}{2} \sin(\beta L) [Si(2\beta L) - 2Si(\beta L)] \right. \\ \left. + \frac{1}{2} \cos(\beta L) [C + \ln(\beta L/2) + Ci(2\beta L) - 2Ci(\beta L)] \right\}$$

where  $C = 0.5772$  (Euler's Constant)

$$Ci(x) = - \int_x^\infty \frac{\cos y}{y} dy = \int_0^x \frac{\cos y}{y} dy$$

$$Si(x) = \int_0^x \frac{\sin y}{y} dy$$



53

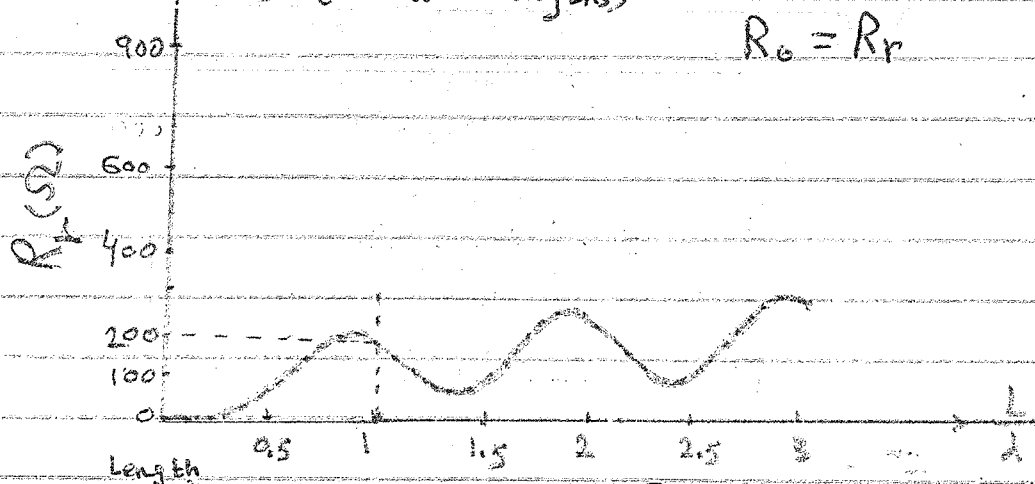
$$C_{in} = \ln(YX) - C_i(X) = \ln Y + \ln X - C_i(X) \\ = 0.5772 + \ln(X) - C_i(X)$$

$$Y = e$$

$$C = 0.5772$$

$C_i$ ,  $S_i$  and  $C_{in}$  are tabulated values

Shown in figure below is a plot of  $R_o$  ( $R_r = R_o$ ) as a function of  $L$  (in wavelengths).



$$R_o = R_r$$

For half-wave dipole ( $L = \frac{\lambda}{2}$ )  $R_o = 73.1 \Omega = R_r$

For Wave Length dipole ( $L = \lambda$ )  $R_o = 200 \Omega = R_r$ .

Directivity of Symmetrical dipole

$$D = \frac{4\pi f(\theta, \phi)_{\max}}{\iint f(\theta, \phi) d\Omega} = \frac{4\pi f(\theta, \phi)_{\max}}{\int_0^{2\pi} \int_0^\pi f(\theta, \phi) \sin\theta d\theta d\phi}$$

$$= \frac{4\pi F(\theta, \phi)_{\max}}{\int_0^{2\pi} \int_0^\pi F(\theta, \phi) \sin\theta d\theta d\phi}$$

$$P_n = \frac{U}{U_m} = \frac{S}{S_m} = \frac{f(\theta, \phi)}{f_m(\theta, \phi)}$$

$$F(\theta, \phi) = F(\theta) = \frac{\cos^2(\beta L \cos\theta/2) - \cos\beta L/2}{\sin\theta}$$

$$= \frac{F(\theta, \phi)}{F_m(\theta, \phi)}$$

$$F(\theta, \phi)_{\max} = F(\theta)_{\max} = \frac{1 - \cos\beta L/2}{1} \quad [\theta = 90^\circ]$$

Directivity

54

## Input impedance

$$Z_A = Z_{in} = R_{in} + j X_{in}, \quad R_{in} = R_A, \quad X_{in} = X_A$$

Input impedance was defined as "the ratio of the voltage to current at a pair of terminals ~~with the ratio~~".

The real part of the input impedance was defined as the input resistance which for a lossless antenna reduces to the radiation resistance, a result of the radiation of real power.

$$\frac{|I_{in}|^2}{2} R_{in} = \frac{|I_0|^2}{2} R_r$$

or

$$R_{in} = \left[ \frac{I_0}{I_{in}} \right]^2 R_r$$

where

$R_{in}$  - <sup>input</sup> ~~radiation~~ resistance at input (Feed) terminals

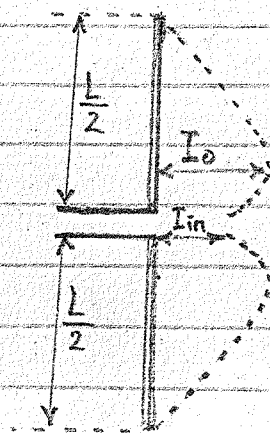
$R_r$  - radiation resistance at current maximum

$I_0$  - current maximum

$I_{in}$  - current at input terminals

$$I_{in} = I_0 \sin\left(\frac{\beta L}{2}\right)$$

$$R_{in} = \frac{R_r}{\sin^2\left(\frac{\beta L}{2}\right)}$$

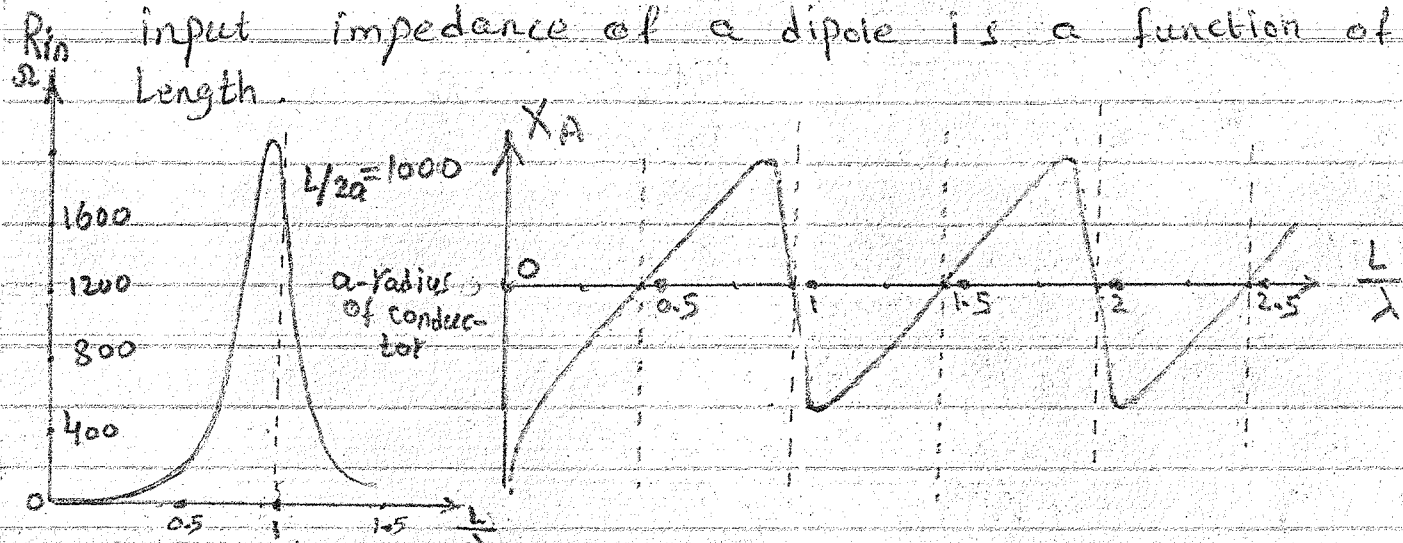


In last two equations the current distribution is ideal and do not account for the finite radius of the wire or the gap spacing at the feed.

$$R_{in} = (R_r + R_l) / \sin^2\left(\frac{\beta L}{2}\right) \quad R_l \text{ - losses resistance}$$

55

The imaginary part (reactance) associated with the input impedance of a dipole is a function of its length.



For half-wave dipole (thin dipole):  $Z_A = 73 + j42.5 \Omega$

$a = \text{radius of antenna conductor (wire)}$

$$D = 2a$$

$$Z_A = Z_{in}$$

Half-wavelength dipole

- Radiation resistance

$$R_o = 60 \left\{ 0.5772 + \ln\left(\frac{2\pi L}{\lambda}\right) - \text{ci}\left(\frac{2\pi L}{\lambda}\right) + \frac{1}{2} \sin\left(\frac{2\pi L}{\lambda}\right) \left[ \text{si}\left(2\frac{2\pi L}{\lambda}\right) - 2\text{si}\left(\frac{2\pi L}{\lambda}\right) \right] \right. \\ \left. + \frac{1}{2} \cos\left(\frac{2\pi L}{\lambda}\right) \left[ 0.5772 + \ln\left(\frac{2\pi L}{\lambda}\right) + \text{ci}\left(2\frac{2\pi L}{\lambda}\right) - 2\text{ci}\left(\frac{2\pi L}{\lambda}\right) \right] \right\} \\ = 73 \Omega$$

- Directivity

$$D = \frac{4\pi}{\int_0^{2\pi} \int_0^\pi P_n(\theta, \phi) d\Omega} = \frac{4\pi}{\int_0^{2\pi} \int_0^\pi P_n(\theta, \phi) \sin\theta d\theta d\phi}$$

$$P_n(\theta, \phi) = \frac{f(\theta, \phi)}{f_{\max}(\theta, \phi)} = \frac{F(\theta, \phi)}{F_{\max}(\theta, \phi)} = \frac{\cos^2[(\pi/2) \cos\theta]}{\sin^2\theta} / 1, \quad F_{\max} = 1$$

$$D = \frac{4\pi}{2\pi \int_0^\pi \frac{\cos^2[(\pi/2) \cos\theta]}{\sin^2\theta} \sin\theta d\theta} = 1.64$$

$$L = 3 \lambda/2$$

$$Z_{in} = 105.49 + j45.54 \quad \Omega$$

} thin dipole



55

Example

Find the power density in  $\text{W/m}^2$ , at a distance of 25 km from an antenna that is radiating 5 kW with a directivity of 36 dB.

Solution

$$G = 36 \text{ dB}$$

$$36 = 10 \log G$$

$$G = 10^{3.6}$$

$$G = 10^{3.6}$$

The power density at the given distance will be

$$W_d = \frac{P G}{4\pi R^2} = \frac{5 \times 10^3 \times 10^{3.6}}{4\pi \times 25 \times 10^3} = 2.5344 \times 10^{-3} \text{ W/m}^2$$

Example

An electric field strength of  $10 \mu\text{V/m}$  is to be measured at an observation point  $\theta = \pi/2$ , 500 km from a half-wave (resonant) dipole antenna operating in air at 50 MHz.  $R_{rad} = 73 \Omega$ ,  $Z_{in} = 73 + j42.5 \Omega$

- What is the length of the dipole?
- Calculate the current that must be fed to the antenna
- Find the ~~average~~ power radiated by the antenna
- If a transmission line with  $Z_0 = 75 \Omega$  is connected to the antenna, determine the standing wave ratio



55

Solution

a) The wavelength  $\lambda = \frac{c}{f} = \frac{3 \times 10^8}{50 \times 10^6} = 6\text{m}$

Hence, the Length of the half-dipole is  $L = \frac{\lambda}{2} = 3\text{m}$

b)

$$|E_0| = \frac{60 I_0}{r} \frac{\cos(\frac{\pi}{2} \cos \theta)}{\sin \theta}$$

$$I_0 = \frac{|E_0| r \sin \theta}{60 \cos(\frac{\pi}{2} \cos \theta)} = \frac{10 \times 10^{-6} \times 500 \times 10^3 \times \sin 90}{60 \cos(90 \cos 90)} = \frac{5}{60}$$

$$[I_0 = I_{in} \text{ for } 1/2 \text{ dipole}] = 83.33 \text{ mA}$$

c)  $P_{rad} = \frac{1}{2} I_0^2 R_{rad} = \frac{1}{2} \times \left(\frac{5}{60}\right)^2 \times 73 = 253.5 \text{ mW}$

d)  $\rho = \frac{Z_L - Z_0}{Z_L + Z_0} \quad (Z_L = Z_{in} \text{ in this case})$

$$= \frac{73 + j42.5 - 75}{73 + j42.5 + 75} = \frac{-2 + j42.5}{148 + j42.5} = \frac{42.55 / 92.69^\circ}{153.98 / 16.02^\circ}$$

$$= 0.2763 / 76.67^\circ$$

$$SWR = \frac{1 + |\rho|}{1 - |\rho|} = \frac{1 + 0.2763}{1 - 0.2763} = 1.763$$